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Can an interest-free credit facility be more efficient than a usurious payday loan?

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Can an interest-free credit facility be more efficient than a usurious payday loan?

Abstract: Inefficiencies in mainstream credit markets have pushed selected households to frequent high cost payday loans for their liquidity needs. Ironically, despite the prohibitive cost there is still persistent demand for the product. This paper rides on the public policy objective of expanding affordable credit to rationed households. Here, we expound a simple model that integrates inexpensive interest-free liquidity facility within an endogenous leverage circuit. This builds on the technology of ROSCA/ ASCRA/ mutual/ financial cooperative and cultural beliefs indoctrinated in Islam. Our results indicate the potential Pareto-efficiency of this interest-free circuit in contrast to the competing interest-bearing schemes of payday lenders and mainstream financiers.

JEL codes: D14, G29, G32, 016, Z12

Keywords: interest-free loan, payday loan, financial exclusion, liquidity facility, cooperatives

1. Introduction

"*Many people, particularly low-to-moderate income households, do not have access to mainstream financial products such as bank accounts and low-cost loans. Other households have access to a bank account, but nevertheless rely on more costly financial service providers for a variety of reasons. In addition to paying more for basic transaction and credit financial services, these households may be more vulnerable to loss or theft and often struggle to build credit histories and achieve financial security*".

FDIC (2009, p.10)

A survey by the Federal Deposit Insurance Corporation (FDIC) in 2009 carries concerns on the extent of financial rationing faced by American households.[1] According to the FDIC (2009), approximately 17.9% or 21 million households who do have banking accounts subscribe to the services of alternative financial service providers. With respect to their credit needs, these households have had to frequent these service providers, including payday loans. In a separate study, Lawrence and Elliehausen (2008) find 73% of the surveyed payday loan borrowers suffered rejection or limitation on their credit application (i.e., rationed or completely rationed out) by mainstream financiers, which is three times above the United States general population. The use of payday loans are largely for unplanned events that highlights the liquidity constrained status of this cohort.

Payday loans or cash advances, are structured to function as a short term liquidity facility to smooth inter-temporal income shocks. This involves issuance of single, small, short-term and unsecured consumer loan, ranging from $100 to $500. An average payday loan is for less than $300, with repayment period of 7 to 30 days (Lawrence and Elliehausen, 2008). The industry has been severely criticised for its high credit cost, in combination with wider issues of predatory practices and expropriation of wealth (OFT, 2013).[2] Undergirding these criticisms is the interest servicing burden (Melzer, 2011) faced by these households who are in the moderate to low income bracket, and lack financial sophistication (Lawrence and Elliehausen, 2008). The fees reflect the industry's severe default rates (DeYoung and Phillips, 2009).[3] Interestingly, despite

1 A further 5 million households may potentially face similar constraints but have been omitted from the above due to paucity of data on their usage of alternative financial services (FDIC, 2009).

2 Predatory lending is characterised by "*excessively high interest rates or fees, and abusive or unnecessary provisions that do not benefit the borrower*" (Carr and Kolluri, 2001, p.1).

3 The industry's default rate of 21% is extremely risky compared to the 3% rate experienced by commercial banks (DeYoung and Phillips, 2009). We find that the high cost concurs with credit literature to compensate for risk associated with these risky borrowers.

heavy criticisms, there is still persistent demand for the products. Thus, this highlights a pressing need to explore inexpensive financial alternatives to assuage the liquidity needs of this market segment.[4] The fact that these households have had to exhaust other credit avenues alludes to the rationed out effect and potentially non-Pareto efficient solution. To date, studies on payday loans have either focused on (i) credit behaviours; or (ii) welfare effect of the borrowers, without delving on Pareto optimal substitutes.

Recognising this shortcoming, the primary motivation of this paper is to expound an institutional design for the provision of inexpensive, short-term liquidity facility, which satisfies the latent demand of these households to smooth their inter-temporal exogenous income shocks. Specifically, our study aims to explore the following question: Can an *endogenous* interest-free payday loan circuit provide a more efficient credit solution in contrast to current payday lenders and mainstream financiers? This is achieved through integrating the two strands of literature on: (i) institutional structures related to endogenous circuits; with (ii) cultural beliefs (i.e., Islamic tenets) in particular, interest-free loans.[5, 6] Our research motivation is consistent with that of Coase (1937) and Alchian (1950), who in their seminal papers rationalise efficient institutions as those that evolve and adapt to the environment to deliver services in a cost effective manner. Moreover, the approach taken in this paper to intertwine institutional design with culture is reflective of Acemoglu et al. (2005, p.424), who reiterate "*belief differences clearly do play a role in shaping policies and institutions*".

For the purpose of this paper, the target population are economically active households. This is consistent with the underwriting criteria of payday lenders that require borrowers to be in employment and bank account holders, as well as with the findings of the FDIC (2009) survey.

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4 Although we have used the United States as the primary reference base, this does not preclude the existence of payday lending in other developed and developing economies.

5 Forms of endogenous circuits include informal institutions of Rotating Savings and Credit Associations (ROSCA) and Accumulating Savings and Credit Association (ASCRA), where members contribute periodically an amount of funds to a common pool over a specified period. In ROSCA, the assignment of the pooled funds to each member is determined either (i) on *random* basis whereby the sequence is only known ex-post to the member at the point of disbursement; (ii) through a *bidding* process to the winning member who pledges higher contribution to the pot or one-time side payment to the other members; or (iii) *fixed*/ pre-determined ex-ante by the ROSCA governing authorities. By pooling resources, it permits the mobilisation of funds that otherwise would have been kept out of circulation. Whilst ASCRA shares similar features of its nemesis, there is greater flexibility in the amount and timing of each member contribution, larger membership, allocation of the pooled funds, and its greater social function (Bouman, 1995). The motives for participating in ROSCA/ ASCRA ranges from savings mechanism to acquire durables, fund life-cycle events, self-control commitment device, insurance and investment avenue of surplus funds to either protect against social/ marital pressures or generate returns (Besley, 1993; Bouman, 1995; Dagnelie and LeMay-Boucher, 2012). Mutual and financial cooperatives are the more advanced and formal forms of these circuits.

6 Charitable concept of interest-free funding is also present in other Abrahamic faiths. For example, the existence of Jewish free loan societies is linked to the obligation in Judaism for extending free loans to the poor (Lewinson, 1999). The integration of Islamic cultural beliefs in the design of this liquidity facility exemplifies its universality in 'democratisation of finance' to the masses.

Additionally, our model is based on risk neutral economic agents.[7] We illustrate the above through an institutional structure of an endogenous leverage circuit formed from member based contributions.[8] This is followed by two stepped extensions that assimilate real world elements of having fraction of borrowers within a finite life circuit, and subsequently extending the circuit as a going concern with random repetitive borrowing. The objective of the basic framework and the extensions are to solve for Pareto-efficiency by simultaneously (i) ensuring availability of affordable credit (where credit is due); and (ii) moderating their commitment issues that promotes long-term financial security. This is showcased by mathematically modelling a short term interest-free liquidity facility circuit that moderates adverse selection and moral hazard. The beauty of the model lies in the structuring of the circuit, where members help one another to alleviate inter-temporal liquidity shocks such that the benefits of borrowing outweigh the cost of it. This draws from the 'barn raising' practices in the United States frontiers discussed in Besley et al. (1993) and captures Commons's viewpoint (1931, p.651), where he states "... *collective action is more than control and liberation of individual action-it is expansion of the will of the individual far beyond what he can do by his own puny acts*".[9]

This paper contributes to existing literature from four perspectives. First, it averts expropriation of wealth of these households through establishing an alternative recourse for liquidity funding. This is in contrast with liquidity stripping from onerous interest charges of current payday loans. It conjointly satisfies public policy call for expansion of affordable credit. Second, our framework allows for satisfaction of liquidity needs of households as solution to rationing by mainstream financiers. Third, we integrate interest-free loans in our liquidity facility. This is drawn from charitable teachings, specifically from Islamic religious tenets that are proffered as a remedy to the prohibited interest (*ribā an-nasi'ah*). Thus, it unveils the economic potential of this antiquated financing, conceived from cultural ideals, as a financial development device. Fourth, by binding eligibility to the liquidity facility with a member's

7 The paper adopts a simple framework of risk neutrality to derive close form solutions. The model can be extended to risk-averse agents by incorporating higher opportunity cost of capital or discount rate 'γ' that comprises an imputed return 'r' (see equation (3) in Section 4). However, we have chosen not to incorporate risk aversion as the resultant outcome only increases the threshold that the circuit needs to observe to ensure fulfilment of the Pareto-efficiency conditions, leaving its fundamentals unaffected. Moreover, this would limit financial participation contrary to the injunction of the *Qur'ān* (the Holy Book of Islam verse 30:39) which prefers charity over exorbitant cost of funding especially for the underprivileged. Our approach is also consistent with Ebrahim (2009).

8 We employ a generic term 'circuit' to signify all institutions where the principal and agent are the same individual. The structure is akin to that of a non-profit institution. An administrator may be present but is not incentivised by rent-seeking motives.

9 Our model reiterates the significant developmental role of endogenous circuits in the 19th century. These circuits permit greater latitude to grant its customers affordable credit compared to profit-oriented mainstream financiers. Recently, the economic importance of endogenous circuits in the United Kingdom was further boosted by the legislative reforms that enabled these institutions (i) greater market reach; and (ii) flexibility to determine its member incentive structures (see HM Treasury, 2012).

fulfilment of the periodical contributions ruling, it harnesses the commitment technology sacrosanct with endogenous leveraged circuit-based institutions.[10] This effectively moderates the issue of time-inconsistent preferences closely associated with payday loan borrowers as well as shelters them from liquidity gaps arising from exogenous shocks.

The remaining of this paper is structured as follows. Section 2 details the landscape of the payday lending industry and related literature. Section 3 discusses the rationale for the prohibition of interest and its contrast against charitable modes in Islamic tenets. In Section 4, we develop a simple model to illustrate the Pareto-efficiency of endogenous interest-free payday loan circuit in addressing financial constraints of these households and its results. Finally, we conclude in Section 5.

2. Landscape of payday industry and related literature

Payday lending emerged in early 1990s in response to increased demand for short term credit following the spatial void created by withdrawal of mainstream banks from small loans, low profit margin business segment (OFT, 2010). The convenience of fast disbursement, minimal or non-existent credit checks further adds to its attractiveness (FDIC, 2009). An indicator of its growth pace is the extensiveness of payday loan network across the United States. Payday lenders have more branch presence then McDonalds and Starbucks combined (Zinman, 2010). Based on the 2009 FDIC survey, approximately four million households have frequented payday lenders, which is now a $38.5 billion industry (FDIC, 2009; CFSA, 2011).[11]

Payday loan customers must be employed and banked to subscribe to these services and according to a survey by Lawrence and Elliehausen (2008) majority are in the moderate income bracket of $25,000 to $49,999. The subscribers are mostly young, below the age of 45, married or living with a partner and having children below the age of 18 years. They justify these households' fits the life-cycle stage where credit demand is high.

10 Dagnelie and LeMay-Boucher (2012) provide empirical evidence on the use of ROSCA as a commitment device that binds households financial conduct from unnecessary spending and protect the savings against theft, losses or social pressures that dissipates the saved amount. Whilst the demand for ROSCA and ASCRA is largely for planned endogenous events, Bouman (1995) does state that members are impelled to participate in these endogenous circuits to safeguard against emergency expenses arising from illness and other misfortunes. In ASCRA, the insurance element is met through the disbursement of its accumulated loans. Unlike ROSCA and ASCRA where the member is required to make compensating payments (i.e. higher payment to compensate other members in a bidding ROSCA or interest on the loaned amount in ASCRA), our Model 3 (see Section 4) provides similar relief without the additional financial burden.

11 In the United Kingdom, the payday loan market is estimated to be worth at £2 billion–£2.2 billion in 2011/2012, with three players controling 57% of the total market loan value (OFT, 2013).

The survey by Lawrence and Elliehausen (2008) find majority are infrequent users of the payday lending facility. However, there is selected few; accounting for 22.5% of total surveyed who have 14 or more loans in the same year. These frequent users tend to rollover the outstanding loan. Generally, these loans would run for 2 weeks or less, or over a 3–4 week periods. These frequent borrowers are more likely to have exposure to more than one payday loan, exhibiting the classic case of borrowing from Peter to pay Paul, where a loan drawn on a new payday lender is often used to offset against an old one.

The primary complaints against payday lenders are the exorbitant finance charge. Fees for a $100 loan ranges from $15 to $30, with annual percentage rate (APR) of 20%–300%. The extremely high cost in contrast to other near credit substitutes raises criticism from consumer advocates and public agencies. According to industry players, the APR is resultant from the small loan size, given payday lenders' high default rates. Industry players argue that the $15 charge is definitely lower than the $50 flat rate returned check fees or a $25 covered overdraft (overdraft protection) by depository institutions (Morgan et al., 2012).

Payday lending is a regulated industry. It is subjected to state and federal laws and some players also subscribe to industry standards of the Community Finance Services Association (CFSA); an industry self regulatory organisation (Lawrence and Elliehausen, 2008).[12] The United States established a new regulatory body, the Consumer Financial Protection Bureau in July 2011 to oversee matters related to consumer protection, including market conduct of payday loan industry. This independent body is part of the financial reforms outlined in the Dodd-Frank Wall Street Reform and Consumer Protection Act 2010.[13]

Studies on payday lending have primarily centred on two aspects; namely consumer credit behaviours and welfare effects on the availability or withdrawal of this credit. Skiba and Tobacman (2008) seek to rationalise the demand for payday loans despite its excessively high fees. They find that payday loan borrowers exhibit partially naive quasi hyperbolic discounting tendencies. In that, the borrowers demonstrate overly optimistic forecast of future outcomes in respect of their own time preference, or their probability of absorbing future shocks.

In a different study, Agarwal et al. (2009) find that the sampled population choose payday loans despite having unused liquidity on their credit cards. This exemplifies existence of liquid

12 The industry is subjected to: (i) the Truth in Lending Act at the federal level that governs disclosure requirements; (ii) Fair Debt Collection Act that regulates debt collection practices; and (iii) National Bank Act that essentially allows the payday lender to enter into rent-a-bank model, which is now defunct by virtue of the stricter FDIC regulation on national chartered banks. The CFSA provides industry best practices that are essentially focused on consumer protection.

13 The governing act for payday loan in the United Kingdom is the Consumer Credit Act 1974 and the industry is presently regulated by the Office of Fair Trading (OFT).

debt puzzle, whereby individuals undervalue their financial options. This also highlights the individuals' lack of cognitive ability to discern costs across different financial products. Gathergood (2012) points persistent indebtedness to poor financial literacy and self-control problems. In such a case, individuals are more likely to succumb to impulsive consumption. The ease of credit provided by high-cost credit providers including payday loan, further exacerbates this tendency and heighten the likelihood of over-indebtedness. The study supports paternalistic approaches in regulations, i.e., preventing access to credit that pushes consumers to succumb to sub-optimal behaviour.

In regards to its welfare effects, the evidence is still debatable. Morse (2011) and Zinman (2010) to name a few, argue that accessibility to payday loan is welfare enhancing, which is in contrast to Skiba and Tobacman (2009) and Melzer (2011).[14] Using the 1996 natural disaster in California as an event that has widespread economic effect on households, Morse (2011) finds that presence of payday lenders reduces emergency distress and serious criminal incidences. Zinman (2010) finds that restricting access creates deterioration in financial position of Oregon households as opposed to those domiciled in Washington, as the control state. Households in the restricted payday state experienced higher unemployment and reported an overall poor future financial outlook. The negative effect of the regulatory ban on payday loan is worsened by the lack of affordable financial substitutes.[15] On the other hand, Melzer (2011) construct the presence of payday loans impairs the financial welfare of the borrowers due to the debt servicing burden associated with this type of credit.

3. Islamic prohibition of interest and the contrast against charity

Salleh et al. (2012) demonstrate that Islamic prohibition of interest in credit transactions (*ribā an-nasi'ah*) is attributed to the inclination for expropriation of wealth. This occurs if there is inequity in the financial contract, thereby resulting in two equilibrium cases. First, in the case of financial repression, where the real interest rate is negative the lender's assets are expropriated. Second, in case of negative leverage, where the real interest rate is greater than the

14 Similar to Skiba and Tobacman (2009), Morgan et al. (2012) find some corroborative evidence of decline in Chapter 13 bankruptcy filings post payday loan ban. However, the authors opine that this require further examination to affirm its robustness.

15 Thirty three states permit payday lending with rules on payday loan terms including maximum fees, rollovers, loan size, licensing and examination requirements as well as collection procedures for past-due loans. Seventeen states totally prohibit offering of payday loans. Contrary to the United States, the United Kingdom refrains from adopting intrusive regulatory measures, such as stringent price controls or complete ban on the services. The OFT (2010) views such controls as market disruptive.

unleveraged expected return on the asset being financed, then the borrower's assets are expropriated. In the long-run, this creates imbalances or non-sustainable equilibria. When a borrower defaults, this can create a domino effect, given the interconnectedness of credit markets. It effectively amplifies volatility within the financial system and thus precipitates financial fragility, as evidenced in the ongoing financial crisis. In extreme cases of agency costs of debt accruing to high project and default risks, this can lead to autarky or financial exclusion, with adverse impact on the underprivileged. Their study alludes to interest-based financial contracts as being Pareto-inferior (or at best it is Pareto-neutral) to a hybrid form.

Instead, the *Qur'ān* (the Holy Book of Islam) contrasts interest (*ribā*) with that of charity (*sadaqah*) (see verses 2:276-277, 30-39). Charity, as defined in the practice of Prophet Muhammad PBUH (*Sunnah*), is not only concerned with financial forms but also all types of good deeds (Sahih Al-Bukhari Vol. 2, 24:144; Sahih Muslim Vol. 3, 12:2329–2330).[16] Piety through charitable deeds inculcates a sense of brotherhood and advances social welfare. As highlighted by Bremer (2004, p.7), "*Charities ...provided a source of support for institutions and interest groups independent of, and sometimes in opposition, to the state. Islamic charities have historically played an additional role in society, that of promoter of decentralized economic development. ...In this respect, they reflect the blending of religious and secular, the social and economic, that is the key characteristic of the Islamic idea*". From a moral perspective, Ibn Taymiyyah (1951), the great Islamic scholar, argues the element of charity cements social cohesiveness, whilst usury factionalises society.

The *Qur'ān* censures the practice of creditors, who cumulate the amount due for every delay in settlement that leads to further financial hardship on the debtor. Instead, it calls for the creditor to grant respite to the borrower such that, if the creditor were to forfeit the amount owed, this reflects a higher order of virtuousness, and will be rightly rewarded (verse 2:280). Unsurprisingly, the prohibition of interest is also enjoined in the religious books of Islam's sister religions, i.e., Judaism and Christianity (see Cornell, 2006).

Both the *Qur'ān* and *Sunnah* have specific references for assisting the underprivileged. The financial forms of charity can be broadly categorised into *zakāt* (social welfare funds), *waqf* (philanthropic foundations) coupled with *qard* (interest-free loan) or *salaf* (synonymous with interest-free loan). *Zakāt* forms one of the five pillars of Islam and is obligatory on one's wealth for the benefit of the recipients identified in the *Qur'ān* (verse 9:60). Of interest is the specific directive for financial resources to be allotted for the poor and needy. Although *waqf* (*awqāf*,

16 Although Islam enjoins charitable deeds, it prohibits begging, for it is best to be actively employed to uplift one's economic status (Sahih Al-Bukhari Vol. 2, 24:1470–1471; Sahih Muslim Vol. 3, 12:2396, 2400 and 2404).

plural) is not mentioned specifically in the *Qur'ān*, it plays an instrumental role in Islamic civilisation. The earliest records on the practice of *waqf* can be traced to the Ottoman Empire in the eight century (Cizakca, 2000). It is said that these philanthropic foundations were able to financially support the provision of social services in Muslim society at that time, and in turn help address economic disparity. Such practice involved the endowment of privately owned properties for charitable purposes in perpetuity. The revenue generated by the *waqf* is then utilised according to its objects.

Qard signifies the extension of loan to a borrower from one's resources without expectation of gains, whereby the lender forfeits the use of his resources during the loaned period. Such is its prominence that it is ranked higher than charity and even equated as a loan to God himself (verses 2:245; 5:12; 57:11 and 18; 64:17; 73:20).[17] This benevolent loan is also synonymous with *salaf* that connotes the extension of a loan, subject to repayment at a later time (Al-Zuhayli, 2003).

From a *fiqh* (Islamic jurisprudence) perspective, jurists are divided on the rights of the lender on the terms of the loan. Two widely opposing views are that a lender has absolute rights to recall the loan at anytime, whilst others view that it is permissible for the lender to stipulate the loaned period and hence, both contracting parties should abide by it (see Al-Zuhayli, 2003 on the debate by the four major *Sunni* schools of thought). This ambiguity in the *Shari'ah* interpretation can cause adverse repercussions in current financial context that warrants property rights certainty.[18] Underdevelopment of the *fiqh* provides ammunition to critique Islamic law (see Kuran, 2011).

Despite these shortcomings, there is documented evidence where *qard* is deployed as a funding mechanism in modern financial dealings. Ebrahim (2009) finds the well-to-do members of clans in Oman informally granting interest-free loans (*qard*) to their destitute clan members for home purchase. In the same study, he explores the potential of formalising interest-free solutions for long-term real estate financing. Other studies on interest-free structures include Darrat and Ebrahim (1999) who focus on open market operation instrument in a partial equilibrium framework of *qard*-based Malaysian Government Investment Certificates. There are also existing practices such as the National Australia Bank (Australia) no interest loan schemes, Akhuwat (Pakistan) no interest microfinance and JAK Members Bank (Sweden).

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17 Ali (2002) connotes *qard* to "*spending in the cause of God*" (footnote 710, p.245). A benevolent loan does not exempt the borrower from honouring the debt. The severity of non-repayment is highlighted in the *Sunnah* whereby even a martyr who is forgiven for every sin is still bound by his debt (Sahih Muslim Vol. 5, 33:4883–4884).

18 Earliest record on employment of *qard* by Az-Zubair also does not allude to its form and activity in which it was deployed (Sahih Al-Bukhari Vol. 4, 57:3129).

Our model explores alternative platforms for deployment of this form of financing and augments present studies and actual practices mentioned above. Furthermore, the employment of interest-free element emphasised in our paper provides a direct contrast to current payday usurious facility. More importantly, according to religious injunction, if the expected return (r) on funds is gradually restrained to zero (moving from r_e to r'_e and finally 0), the supply of funds (S) will contract to a level where funding disappears (see Figure 1 below). This is the probable reason why scholars like Fazlur Rahman (1964) compartmentalises this form of funding to only philanthropic endeavours. However, this study demonstrates that lending is revived by embedding the interest-free credit facility within a circuit, which promotes group insurance. This is because members help each other when faced by misfortunes (exogenous liquidity shocks).

[Insert Figure 1 here]

4. Model development

This section details the mathematical design of an *efficient interest-free* short term payday loan facility (using *endogenous* leverage) to address the inter-temporal liquidity needs of payday loan borrowers. Our endogenous leveraged circuit is founded in the works of institutional economics (Commons, 1931), and builds from the technology of ROSCA (Besley et al., 1993; Dagnelie and LeMay-Boucher, 2012), its associated hybrids; namely, ASCRA (Bouman, 1995), and the more contemporary mutual and financial cooperative (Ebrahim, 2009). Besides liquidity transformation, the circuit features akin to an Islamic insurance (*takāful*) or mutual scheme where members guarantee each other from unexpected damage, losses or misfortune (Bouman, 1995).

Furthermore, unlike other endogenous leverage groupings, liquidity constrained members of the circuit receive short term interest-free payday loans, which is repaid at their next payday date. Our model expounds the elements that need to be observed if an interest-free loan that is enjoined in Islam is to have a profound impact in any financial development schemes. Here, we demonstrate that this endogenous interest-free payday loan circuit integrated with appropriate constraints that circumvent adverse selection and moral hazard can be Pareto efficient or at least neutral to that of its competitor, i.e., payday lender and mainstream financier.

The interest-free payday loan circuit is structured as follows. Individuals are required to become members by contributing monthly to a common pool of funds, i.e., circuit members. In our model, members are risk neutral, and the demand for liquidity or payday loan is treated as

exogenous. Members can only apply for the interest-free loan, i.e., borrow, after qualifying a defined period of membership. This gestation period has a two-fold effect. First, it allows the circuit to identify and assist the member in realisation of her/his financial goals. Second, it allows member to build up equity cushion through their monthly contributions. This effectively binds the member to the circuit and addresses member time-inconsistent preferences. In addition to these two covenants, other mandatory rulings to address adverse selection and moral hazard issues (i.e., default cost) include requirement for (i) direct deposit of member paycheque into the circuit; and (ii) existence of loan guarantor (see detailed explanation below). Furthermore, once a member borrows from the circuit, she/he is required to undergo financial planning program to enhance her/his financial literacy. This helps errant members to plan ahead, alleviate future liquidity crises and stay debt free.

[Insert Figure 2 here]

A stylised depiction of the models set up is summarised in Figure 2. This comprises of a basic institutional framework and two stepped extensions that embeds real-world practicalities. Model 1 (see Limb A of Figure 2) illustrates the basic structure of our efficient interest-free endogenous leverage circuit across a one period cycle. Here, all circuit members are savers and also liquidity constrained borrowers. This conforms to a self-insurance scheme. The extension to this basic framework is provided in Models 2 and 3 (see again Limbs A and B of Figure 2). In Model 2, we relax the simultaneity in borrowing needs. That is, only fraction of members will borrow to tide their liquidity shortfall. Model 3 further relaxes the elements whereby there is random multi-period borrowing that in the long-run approaches a steady state. As shown in Figure 2 in the case of defaulting borrowers, the circuit retains the accumulated contributions or savings of the defaulters. Otherwise, they receive their savings net of the amount loaned. On the other hand, non-borrowing members are entitled to their savings.

For all three models, we implicitly assume the existence of an information architecture, where property rights needed for the forthcoming paycheque to serve as collateral, accurate methods of verifying or evaluating members' income and bankruptcy procedures are well established (see Levine et al., 2000). Individuals joining the circuit are assumed to have limited asset qualifying collateral and void of other alternative credit solutions, would have to subscribe to current high cost payday loan. Each of the circuit members receives an exogenous flow of income. The above assumptions are representative of the stylised facts of payday loan borrowers' demographics (see Lawrence and Elliehausen, 2008; FDIC, 2009).

4a. Model 1: Institutional basic framework where all members are both savers and liquidity constrained borrowers

[Insert Figure 3 here]

(i) As depicted in Figure 3, each member is required to make periodic monthly contributions '*C*' into a pooled fund, from time $i = -m$ (at the point of membership) to $i = 1$ (the circuit terminal date). Here, we adopt monthly contributions to maintain consistency with members' income stream, i.e., paycheques are generally issued on monthly basis. By instituting periodic contribution, we: (i) alleviate adverse selection, as it reveals the financial status of the prospective borrower through her/his income level (especially during the gestation period) as illustrated in the income constraint (see Equations (5)-(5a) below and also Akerlof, 1970); (ii) assist in long-run accumulation of wealth that minimises exposures to exogenous income shocks; (iii) initiate an equity buffer that minimises the likelihood of the member to strategically default on her/his borrowing (see Foote et al., 2008); and most importantly (iv) implement a commitment device that moderates self-control issues associated with payday loan borrowers (see Skiba and Tobacman, 2008; Dagnelie and LeMay-Boucher, 2012).

We also alleviate moral hazard element by: (i) instituting a loan constraint (see Equations (6), (6a) and (6b) and also Ebrahim, 2009); (ii) implementing a collateral constraint in the form of a co-signer (see also Stiglitz, 1990); and (iii) integrating compulsory financial programs that enhances the financial status of the member requesting funds from the cooperative (see Bernheim and Garret, 2003).

The accumulated periodic contribution, represented by '*S*', forms the capital base of this circuit and is used to meet short term financial needs of liquidity constrained members.[19] This is given in Equation (1), where '*m*' denotes the month building up to the disbursement of the interest-free payday loan facility.

$$S = \sum_{i=-m}^{t} C = C(t + m + 1) \qquad (1)$$

When $t = 1$ (as in Models 1 and 2), this culminates into

19 To assist in the circuit start up and reduce the lag in time to loan disbursement, the circuit may employ seed funding from charitable funds (e.g. *zakāt* and *sadaqah* funds). Ideally for long-run stability, this charitable fund should be institutionalised and performs the central role of providing liquidity relief to individual circuits that may suffer from unforeseen shocks. This is akin to the *Verband*, associative level of the German cooperative banking system (Biasin, 2010).

$$S = \sum_{i=-m}^{1} C = C(m+2) \tag{1a}$$

(ii) After satisfying the minimum gestation period, liquidity constrained members qualify to draw '*Q*' interest-free payday loan from the circuit at time *i = 1-n* (see Figure 3), where '*n*' is a fraction of a month (i.e., $n < 1$ month).[20] The interest-free facility resembles a bullet loan, where total repayment of principal '*Q*' is made at terminal time *i = 1*. The loan repayment is net of the accumulated contributions '*S*'.

$$(Q - S) \tag{2}$$

(iii) To account for the opportunity cost of capital employed within the circuit, '*C*' and '*Q*' are discounted by 'γ'. That is, the monthly discount rate comprises of an imputed return '*r*', which is equivalent to the average cost of fund incurred in mainstream credit market.

$$\gamma = \frac{1}{(1+r)} < 1, \ \forall \, r > 0 \tag{3}$$

(iv) We also incorporate the fractional transaction cost 'ζ' associated with administering the circuit, eg. management of members' contributions and loan processing (Kontolaimou and Tsekouras, 2010), and fraction default 'α' (Jaffee and Russell, 1976).[21, 22] It should be noted that this fractional default 'α' represents the proportion of defaulters (ex-post any recoveries from respective loan guarantors) from the circuit's total population of borrowers. The circuit efficiency is contingent on minimising transaction costs and default, as they can fritter away the circuit's gains or cause erosion to its capital base (Coase, 1937; Alchian, 1950). Both outflows are moderated by presence of covenants discussed below. Additionally, the circuit also retains right of recourse on defaulting borrowers' savings 'αS'.

In line with the circuit's objectives, the discounted 'γ' contributions and interest-free loan after accounting for transaction 'ζ' costs and default 'α', coupled with net loan payoff, given by Equation (4) should at least be equal or greater than zero.

20 This implies members face liquidity problems before their next paycheque.

21 The institutional structure of the circuit already minimises upfront transaction costs compared to current payday lenders, as it: (i) benefits from non-profit motive management force; (ii) does not incur external funding costs; and (iii) is not bound to issue investment returns to its 'depositors'.

22 Intuitively, utility derived from an interest-free credit facility would be higher than subscribing to high cost current payday loans or face credit rationing from mainstream credit. Therefore in such situations, we foresee that the (non) pecuniary costs associated with default penalty should be significantly severe such that it impels repayment of the loan (see Skiba and Tobacman (2008) for empirical evidence of the degree of reliance of these borrowers on payday loans for their liquidity needs).

$$\underbrace{\left[\sum_{i=-m}^{1} C(1-\zeta)(\gamma^{i})\right]}_{\text{Receipt of member monthly contributions}} \underbrace{-Q(1-\zeta)(\gamma^{1-n})}_{\text{Disbursement of interest-free payday loan to liquidity constrained members (borrowers)}} \underbrace{+\gamma[(1-\alpha)(Q-S)}_{\text{Repayment of non-defaulting borrowers' savings net of outstanding loan}} \underbrace{+\alpha S]}_{\text{Retention of defaulting borrowers' savings}} \geq 0 \quad (4)^{23,\ 24}$$

Substituting '*S*' in Equation (1) into Equation (4) gives us:

$$C(1-\zeta)(\gamma^{-m})[1+...+\gamma^{m+1}]-Q(1-\zeta)(\gamma^{1-n})+\gamma[Q(1-\alpha)+C(m+2)(2\alpha-1)]\geq 0 \quad (4a)$$

The periodic contribution in Equation (4a) form a geometric series that can be further simplified as follows:

$$C\left[\frac{(1-\zeta)(\gamma^{-m})(1-\gamma^{m+2})}{1-\gamma}+\gamma(m+2)(2\alpha-1)\right]\geq Q[(1-\zeta)(\gamma^{1-n})-\gamma(1-\alpha)] \quad (4b)$$

$$\Rightarrow Q\leq C\left[\frac{(1-\zeta)(\gamma^{-m})(1-\gamma^{m+2})}{1-\gamma}+\gamma(m+2)(2\alpha-1)\right]\left[\frac{1}{(1-\zeta)(\gamma^{1-n})-\gamma(1-\alpha)}\right] \quad (4c)$$

Potential maximum loan is,

$$Q_{\max 1}=C\left[\frac{(1-\zeta)(\gamma^{-m})(1-\gamma^{m+2})}{1-\gamma}+\gamma(m+2)(2\alpha-1)\right]\left[\frac{1}{(1-\zeta)(\gamma^{1-n})-\gamma(1-\alpha)}\right] \quad (4d)$$

To alleviate the risk of adverse selection and moral hazard, it is imperative for the circuit to institute covenants as follows:

(i) *Income and loan constraint*: Each member is subjected to an after tax income test '*y*' to ascertain her/his capacity to meet her/his periodic contribution and loan obligation. This not only supports responsible lending (Carr and Kolluri, 2001), but also moderates the adverse selection and moral hazard issues (Jaffee and Russell, 1976). The member's financial capacity is represented by a multiple '*b*' of her/his income and loan.

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23 The circuit structure is designed to be contribution and time invariant for each member joining the pooled fund. That is, each member is required to make periodic monthly contributions to the pooled fund '*C*' from the point of membership at time $i = -m$ to the circuit terminal date, $i = 1$ (in the case of Models 1 and 2) and $i = T$ (in the case of Model 3) (refer Equations (1) and (1a)). This similarly applies to the aggregated member contributions. Consequently, this does not affect the outcome of Equation (4).

24 Each term in Equation (4) signifies either a cash inflow (represented by a positive sign) or an outflow (represented by a negative sign). Each of these terms is discounted by 'γ' that comprises an imputed return '*r*', which is equivalent to the average cost of fund incurred in mainstream credit market. This is a standard treatment of discounting in finance to account for the opportunity cost of capital (i.e. next best investment avenue forgone by the members). The first term thus represents the discounted value of member contributions, while the third term represents that of payback of loans disbursed (netted against their aggregate contributions) after adjusting for defaults.

(ia) Income constraint 'b_1'

Here, the income constraint 'b_1' curtails the contribution 'C' and thus mitigates adverse selection. This is simplified as follows:

$$\frac{y}{C} \geq b_1, \text{ which can be rewritten } \Rightarrow C \leq \frac{y}{b_1} \quad (5)$$

$$\Rightarrow C_{\max} = \frac{y}{b_1} \quad (5a)$$

(ib) Loan constraint 'b_2'[25, 26]

Here, the loan constraint 'b_2' curtails the loan amount 'Q' and thus mitigates moral hazard. This is simplified as follows:

$$\frac{y}{Q} \geq b_2, \text{ which can be rewritten } \Rightarrow Q \leq \frac{y}{b_2} \quad (6)$$

$$\text{Here, } Q_{Binding,\ t-n} = \min\left\{\frac{y}{b_2}, Q_{\max,\ t-n}\right\}, \text{ where } Q_{\max,\ t-n} = \min\left\{Q_{\max 1}, Q_{\max 2,\ t-n}\right\} \quad (6a)$$

$Q_{\max 1}$ is defined in Models 1–3 (sections 4a–4c) respectively by Equations (4d), (7e) and (16d), while $Q_{\max 2,\ t-n}$ reflects the resource constraint of the circuit given by $Q_{\max 2,\ t-n} = C(1-\zeta)(t+m)$.

$$\text{If transaction cost is low, i.e., } \frac{y}{b_2} < Q_{\max}, \text{ then } Q_{Binding,\ t-n} = \frac{y}{b_2} \quad (6b)$$

(ii) *Pre-commitment constraint*: Members are subjected to salary deduction to moderate time inconsistent preference tendencies (Skiba and Tobacman, 2008; Dagnelie and LeMay-Boucher, 2012) and moral hazard. With this, it partially limits the member consumption options available in the future. This seamless transfer of member income to the circuit and subsequent settlement of the interest-free payday loan has a secondary effect of lowering transaction costs of the circuit.

(iii) *Collateral constraint*: Given the potential limited ability of these households to raise asset qualifying collateral, disbursement of the interest-free payday loan is then subjected to a reputable co-signer, who provides surety upon default by the member. The co-signer, who has local information compared to the circuit, is in a preferred position to conduct ex-post

25 This is consistent with Ebrahim (2009).

26 The interest-free payday loan facility is strictly for managing inter-temporal liquidity shocks faced by its members. In tandem with this objective, 'Q' should therefore be confined to a reasonable multiple of its members' monthly after tax income. This helps alleviate debt entrapment, discussed in Lawrence and Elliehausen (2008) and OFT (2013). Nonetheless, our model can still be adapted to reflect allowances for this restriction.

monitoring and impose social sanctions (see Stiglitz, 1990).[27] This then, significantly reduces costly state verification issues, particularly in dealings with low net worth members. However, failure of the co-signer to act accordingly can have a detrimental effect on the circuit efficiency/ sustainability (see Guinanne, 1994 on demise of Irish credit union).

(iv) *Financial capability constraint*: Each member who borrows is required to undergo personal finance program (eg. money management, asset building and debt management) to enhance their financial capability (Agarwal et al., 2009; Gathergood, 2012).[28] This non-pecuniary cost of borrowing is an interventionist measure that has its roots in behavioural finance, as it seeks to influence the cognitive psychology of payday borrowers with regards to their financial conduct (see Bernheim and Garrett, 2003 on the positive long-term behavioural effects of increased exposure to financial education).[29]

Proposition 1. For the circuit to be competitive, its net surplus must satisfy the efficiency condition given by Equation (4c) [30]

Equation (4c) signifies three possible states of the circuit. First, when the circuit fulfils the equality sign, the circuit is at best Pareto-neutral to its competitors, namely mainstream financiers.[31] Second, if the inequality sign is satisfied, the circuit is then Pareto-superior to its competitors. The surplus capital signifies welfare improvement of an initially liquidity constrained group. Third, if Equation (4c) is unmet, then the circuit is Pareto-inferior with erosion in its capital base, and its continued sustainability is doubtful. Here, its sustainability is contingent on minimising transaction costs and defaults, as both erode the circuit's gains and ultimately its capital base. Therefore, the circuit administrators must institute controls, so that

27 The collateral covenant should not be a major participation constraint in lieu that members are required to be economically active. The co-signer can be from or outside the circuit. Where the co-signer is also a member of the circuit, co-signing incentivises peer monitoring, in view that the sustainability of the circuit ultimately affects the interest of the co-signer (Stiglitz, 1990).

28 Our model can accommodate the funding for the personal finance program through the transaction costs 'ζ' in the administration of the circuit. This can also be complemented by financial education public policy programs or specific workplace schemes.

29 Other interventionist measures, which are pecuniary in nature, are to gradually: (i) decrease b_1, and (ii) increase b_2, thereby compelling erring borrowers to save and avoid debt entrapment. This can be extended in our model to incorporate real world practicalities.

30 Our analysis is rationalised based on a standard Net Present Value analysis employed in Financial Management (see Brealey et al., 2011). We have not provided a formal proof as it is a normal practice in the field to discount cash flows of alternate ways of funding a project in order to evaluate an efficient scheme. This approach is also adopted by Ebrahim (2009).

31 We can also deduce that the circuit is Pareto-superior to that of contemporary payday loan, in view of the latter's high cost of funds.

both costs are reduced significantly. This is achieved through various covenants and retention of defaulting member savings as highlighted earlier.

4b. Model 2: Impute real-world element by relaxing the borrowing condition in that only a fraction of members 'λ' borrow from the endogenous circuit

[Insert Figure 4 here]

As in Model 1, members are required to contribute '*C*' on monthly basis upon entry, at $i = -m$ to $i = 1$ period. The following similarly hold in Model 2: (i) the circuit has a defined period, i.e., one-period cycle until $i = 1$, after which it terminates; (ii) variables defined in Equations (1), (2), (3) and (4); (iii) transaction costs and default; and (iv) the four covenants (i.e., income, pre-commitment, collateral and financial capability). However, Model 2 specification differs from previous in that it conceives the likelihood of liquidity strained members may occur at different circuit cycles. Therefore, at any one time, there are a fraction of borrowers signified by 'λ' that are supported by '$(1-\lambda)$' lenders or non-borrowers (see Figure 4). This clearly depicts the 'transformation service' provided by the circuit, whereby the temporary idle funds of a proportion of members (lenders or non-borrowers) are used to provide liquidity to others who suffer from exogenous inter-temporal income shocks. This improves on "*competitive market by providing better risk sharing among people who need to consume at different random times*" (Diamond and Dybvig, 1983, p.402).

As with the previous section, observance of '*C*' entitles member a right to draw on the circuit funds if she/he faces liquidity squeeze. We find the technology of the circuit in Model 2 best resembles the practice of mutual or Islamic insurance (*takāful*), where members agree to indemnify each other against a defined loss. Based on the concept of solidarity, members of the group contribute to a specified fund that entitles each person to protection on occurrence of the loss event. The commercial implementation of this concept of mutuality can be traced to the eight century, where sea merchants would initiate a pool to protect themselves against perils during their voyages (Alhabshi and Razak, 2011).[32] A characteristic that differentiates mutual/ Islamic insurance from the mainstream is that, in the former, each member is the insurer and also insured, which means there is risk sharing between members rather than risk shifting.

32 Although there is no direct reference to *takāful* in Islamic scriptures, the concept finds support in the *Qur'ānic* verses and *Sunnah* that call for upholding of brotherhood and solidarity in times of hardship (*Qur'ān* 5:2; Sahih Muslim Vol. 6, 45: 6585–6590, 45:6669–6674).

Based on the above extension (see Figure 4), total borrowings in the circuit are now signified by 'λQ'. At time $i = 1$, i.e., expiry of the circuit cycle, (a) non-defaulting borrowers are required to settle the outstanding interest-free payday loan net of their savings '$\lambda(1-\alpha)(Q-S)$'; and (b) any defaulting borrowers will have their accumulated contributions or savings retained within the circuit '$\lambda\alpha S$'. The proportion of non-borrowing members are then entitled to a payback of their accumulated contributions constituting '$(1-\lambda)S$'.[33] Equation (7) is a modification of Equation (4), as it incorporates the fraction of borrowing and non-borrowing members.

$$\underbrace{\left[\sum_{i=-m}^{1} C(1-\zeta)(\gamma^{i})\right]}_{\text{Receipt of member monthly contributions}} \underbrace{-\lambda Q(1-\zeta)(\gamma^{1-n})}_{\text{Disbursement of interest-free payday loan to liquidity constrained members (borrowers)}} \underbrace{+\gamma[\lambda(1-\alpha)(Q-S)}_{\text{Repayment of non-defaulting borrowers' savings net of outstanding loan}} \underbrace{+\lambda\alpha S}_{\text{Retention of defaulting borrowers' savings}} \underbrace{-(1-\lambda)S]}_{\text{Repayment of non-borrowers' savings}} \geq 0 \qquad (7)$$

$$\Rightarrow \left[\sum_{i=-m}^{1} C(1-\zeta)(\gamma^{i})\right] - \lambda Q(1-\zeta)(\gamma^{1-n}) + \gamma[\lambda Q - \lambda\alpha Q + S(2\lambda\alpha - 1)] \geq 0 \qquad (7a)$$

Substituting 'S' in Equation (1) into Equation (7a) gives us:

$$C(1-\zeta)(\gamma^{-m})[1+.....+\gamma^{m+1}] - \lambda Q(1-\zeta)(\gamma^{1-n}) + \gamma[\lambda Q(1-\alpha) + C(m+2)(2\lambda\alpha - 1)] \geq 0 \qquad (7b)$$

This is further simplified as follows:

$$C\left[\frac{(1-\zeta)(\gamma^{-m})(1-\gamma^{m+2})}{1-\gamma} + \gamma(m+2)(2\lambda\alpha - 1)\right] \geq \lambda Q[(1-\zeta)(\gamma^{1-n}) - \gamma(1-\alpha)] \qquad (7c)$$

$$\Rightarrow Q \leq C\left[\frac{(1-\zeta)(\gamma^{-m})(1-\gamma^{m+2})}{1-\gamma} + \gamma(m+2)(2\lambda\alpha - 1)\right]\left[\frac{1}{\lambda(1-\zeta)(\gamma^{1-n}) - \gamma(1-\alpha)}\right] \qquad (7d)$$

Potential maximum loan is,

$$Q_{\max 1} = C\left[\frac{(1-\zeta)(\gamma^{-m})(1-\gamma^{m+2})}{1-\gamma} + \gamma(m+2)(2\lambda\alpha - 1)\right]\left[\frac{1}{\lambda(1-\zeta)(\gamma^{1-n}) - \gamma(1-\alpha)}\right] \qquad (7e)$$

Proposition 2. The efficiency condition of the circuit with fraction of members who are borrowers is contingent on satisfaction of Equation (7d)

33 In our model, a member's primary objective in joining the circuit is to ensure access to low cost credit, i.e., maximise borrowing opportunity, in contrast to high cost credit from payday lenders or financial rationing. Given the above motivation, we have not incorporated dividends or investment returns on the accumulated contributions (savings) as these may be better served by existing financial intermediaries.

The three efficiency states described earlier in Proposition 1 apply in Proposition 2. This is even with the added complexity, where not all members will borrow during the same circuit cycle. The circuit can ensure that it satisfies the inequality sign in Equation (7d) by enhancing its predictive ability on probability of liquidity calls by its members. This is closely associated with the principle law of large numbers employed in insurance pricing. By collating sufficiently large number of exposures, the randomness in the occurrence of the exposures will statistically converge towards a defined mean with a given variance, which then allows insurers to fairly predict the frequency and severity of their exposures and price the insurance products accordingly. In the case of the circuit, it can then correctly determine the loan amount and tenure that is feasible to limit liquidity gaps at the end of the circuit cycle.

4c. Model 3: Extends further the real-world element whereby there is random borrowing over multi-periods that in the long-run approaches a steady state

[Insert Figure 5 here]

We further extend the model to allow for multi-period endogenous leverage, where members pool their endowments across time, in order to assure accessibility to short-term interest-free payday loan, in light of unexpected contingencies (see Figure 5). This brings the circuit nearer to that of contemporary financial cooperatives (Ebrahim, 2009). Here, we have a random process of member borrowing. This discrete-parameter Markov chain of $\{X_{t-n},\ t>n\}$ is represented by:

$$P(X_{t+1-n} = j / X_{1-n} = i_1, X_{2-n} = i_2, ..., X_{t-n} = i_t) = P(X_{t+1-n} = j / X_{t-n} = i_t) \qquad (8)$$

Equation (8) essentially assumes a member's future borrowing behaviour is a consideration of only her/his present behaviour, and is independent of the member's past history. The initial probability vector 'p_{t-n}' is denoted by probability of borrowing 'λ_{t-n}' and non-borrowing '$1\text{-}\lambda_{t-n}$', respectively:

$$p_{t-n} = \begin{bmatrix} \lambda_{t-n} & 1-\lambda_{t-n} \end{bmatrix} \qquad (9)$$

We also assume during the next interval that there is a probability 'ρ_{t+1-n}' that members borrow and '$1\text{-}\rho_{t+1-n}$' otherwise. The two-state Markov chain transition probability matrix is illustrated below.

$$P = \begin{array}{c} \\ Borrow_{t-n} \\ NoBorrow_{t-n} \end{array} \begin{array}{c} \begin{array}{cc} Borrow_{t+1-n} & NoBorrow_{t+1-n} \end{array} \\ \begin{bmatrix} \rho_{t+1-n} & 1-\rho_{t+1-n} \\ \phi_{t+1-n} & 1-\phi_{t+1-n} \end{bmatrix} \end{array} \tag{10}$$

The above two-state transition matrix converges in steady state as follows (see Hsu, 2011).

$$P = \begin{bmatrix} \rho & 1-\rho \\ \phi & 1-\phi \end{bmatrix} \tag{10a}$$

This matrix in Equation (10a) is further simplified using the well-known Bayes' rule, as illustrated in the Appendix, where we realise $\phi = \frac{\lambda(1-\rho)}{1-\lambda}$, and $1-\phi = \frac{1-2\lambda+\rho\lambda}{1-\lambda}$.

$$\Rightarrow P = \begin{array}{c} \\ Borrow_{t-n} \\ NoBorrow_{t-n} \end{array} \begin{array}{c} \begin{array}{cc} Borrow_{t+1-n} & NoBorrow_{t+1-n} \end{array} \\ \begin{bmatrix} \rho & 1-\rho \\ \frac{\lambda(1-\rho)}{1-\lambda} & \frac{1-2\lambda+\rho\lambda}{1-\lambda} \end{bmatrix} \end{array} \tag{10b}$$

The long-run borrowing behaviour converges to a steady state '$\hat{\rho}$'. That is, there exists a stationary distribution for the Markov chain. This is found by solving

$$\hat{p}\begin{bmatrix} \rho & 1-\rho \\ \frac{\lambda(1-\rho)}{1-\lambda} & \frac{1-2\lambda+\rho\lambda}{1-\lambda} \end{bmatrix} = \hat{p} \tag{11}$$

Where, $\hat{p} = [s_1 \quad s_2]$, and $s_1 + s_2 = 1$ (12)

Equation (11) can then be rewritten as follows:

$$[s_1 \quad s_2]\begin{bmatrix} \rho & 1-\rho \\ \frac{\lambda(1-\rho)}{1-\lambda} & \frac{1-2\lambda+\rho\lambda}{1-\lambda} \end{bmatrix} = [s_1 \quad s_2] \tag{13}$$

Solving the matrix, we obtain two equations described below:

Equation 1

$$s_1\rho + s_2\frac{\lambda(1-\rho)}{1-\lambda} = s_1 \tag{14}$$

$$\Rightarrow s_1 = s_2\left(\frac{\lambda}{1-\lambda}\right), \ \forall\, \rho \neq 1 \tag{14a}$$

Equation 2

$$s_1(1-\rho)+s_2\left(\frac{1-2\lambda+\rho\lambda}{1-\lambda}\right)=s_2 \tag{15}$$

$$\Rightarrow s_1=s_2\left(\frac{\lambda}{1-\lambda}\right),\ \forall\ \rho\neq 1 \tag{15a}$$

Thus, both Equations (14a) and (15a) lead to the same solution, implying the exogeneity of 'ρ'. By substituting 's_1' in Equation (14a) into Equation (12), we get

$$1-s_2=s_2\left(\frac{\lambda}{1-\lambda}\right) \tag{15b}$$

$$\Rightarrow 1=s_2\left(\frac{1}{1-\lambda}\right)$$

$$\Rightarrow s_2=1-\lambda\text{, and hence } \Rightarrow s_1=\lambda \tag{15c}$$

Thus, restating '$\hat{\rho}$' of Equation (12) with the results derived in Equation (15c) gives us the steady state matrix as follows:

$$\hat{\rho}=\begin{bmatrix}\lambda & 1-\lambda\end{bmatrix} \tag{15d}$$

Proposition 3. A member borrowing behaviour is contingent on her/his past borrowing history.

We find member borrowing behaviour is path dependent, which corroborates the empirical evidence documented in Lawrence and Elliehausen (2008). Despite this intricate issue of path dependency, we can still determine the loans to be underwritten by exploiting the property of steady state, where a fraction 'λ' of the population borrow (irrespective of previous borrowing). For mathematical tractability and aligned with Lawrence and Elliehausen (2008), we assume borrowers who do not redeem their loans would continuously rollover their facility. Therefore, default emerges only at terminal period 'T' (see Figure 5 and Equation (16)). All other variables and covenants remain the same.

$$\underbrace{\left[\sum_{i=-m}^{T}C(1-\zeta)(\gamma^i)\right]}_{\text{Receipt of member monthly contributions}}-\underbrace{\left[\sum_{i=t-n}^{T-n}\lambda Q(1-\zeta)(\gamma^i)\right]}_{\text{Disbursement of interest-free payday loan to liquidity constrained members (borrowers) over multi-period cycle}}+\underbrace{\left[\sum_{i=t+1-n}^{T-1}\lambda Q\gamma^i\right]}_{\text{Loan settlements by borrowers over a multi-period cycle}}$$

$$+\gamma^{T}\underbrace{\left[\lambda(1-\alpha)(Q-S)\right.}_{\text{Repayment of non-defaulting borrowers' savings net of outstanding loan}}+\underbrace{\lambda\alpha S}_{\text{Retention of defaulting borrowers' savings}}-\underbrace{\left.(1-\lambda)S\right]}_{\text{Repayment of non-borrowers' savings}}\geq 0 \tag{16}$$

We substitute 'S' from Equation (1) into Equation (16) and simplify it to derive

$$C(1-\zeta)(\gamma^{-m})\left[1+\ldots+\gamma^{T+m}\right]-\lambda Q(1-\zeta)(\gamma^{t-n})\left[1+\ldots+\gamma^{T-t}\right]+\lambda Q\left(\gamma^{t+1-n}\right)\left[1+\ldots+\gamma^{T-t-2+n}\right]$$
$$+\gamma^{T}\left[\lambda Q(1-\alpha)+\lambda C(T+m+1)(2\lambda\alpha-1)\right]\geq 0 \tag{16a}$$

$$\Rightarrow C(1-\zeta)(\gamma^{-m})\left(\frac{1-\gamma^{T+m+1}}{1-\gamma}\right)-\lambda Q(1-\zeta)(\gamma^{t-n})\left(\frac{1-\gamma^{T-t+1}}{1-\gamma}\right)+\lambda Q\left(\gamma^{t+1-n}\right)\left(\frac{1-\gamma^{T-t-1+n}}{1-\gamma}\right)$$
$$+\gamma^{T}\left[\lambda Q(1-\alpha)+\lambda C(T+m+1)(2\lambda\alpha-1)\right]\geq 0 \tag{16b}$$

$$\Rightarrow Q\leq C\left[\left(\frac{(1-\zeta)\left(\gamma^{-m}\right)\left(1-\gamma^{T+m+1}\right)}{1-\gamma}\right)+\gamma^{T}\lambda(T+m+1)(2\lambda\alpha-1)\right]\text{X}$$
$$\left[\left(\frac{1-\gamma}{\lambda\left[(1-\zeta)\left(\gamma^{t-n}\right)\left(1-\gamma^{T-t+1}\right)-\left(\gamma^{t+1-n}\right)\left(1-\gamma^{T-t-1+n}\right)\right]}\right)-\left(\frac{1}{\gamma^{T}\lambda(1-\alpha)}\right)\right] \tag{16c}$$

Potential maximum loan is,

$$Q_{\max 1}=C\left[\left(\frac{(1-\zeta)\left(\gamma^{-m}\right)\left(1-\gamma^{T+m+1}\right)}{1-\gamma}\right)+\gamma^{T}\lambda(T+m+1)(2\lambda\alpha-1)\right]\text{X}$$
$$\left[\left(\frac{1-\gamma}{\lambda\left[(1-\zeta)\left(\gamma^{t-n}\right)\left(1-\gamma^{T-t+1}\right)-\left(\gamma^{t+1-n}\right)\left(1-\gamma^{T-t-1+n}\right)\right]}\right)-\left(\frac{1}{\gamma^{T}\lambda(1-\alpha)}\right)\right] \tag{16d}$$

Proposition 4. The efficiency condition of a circuit with borrowing by a fraction of members across multi-periods is contingent on satisfaction of Equation (16c)

The efficiency states detailed in Proposition 1 similarly applies for Proposition 4. We find that the circuit's efficiency can be improved in a multi-period model. A circuit that is conducted repeatedly over a series of periods will have greater latitude on its borrowing policy, as each borrower's financial conduct is fully revealed (Hosios and Peters, 1989). By instituting renewal model that is dependent on the member's financial conduct, the circuit effectively addresses conflict of interest between borrowers and non-borrowers. In this situation, each member will

endeavour to undertake fewer risks, which would potentially affect access to future liquidity facility. Credible threat of sanctions in multi-period states can also reduce moral hazard (Stiglitz, 1990).

4c(1). Numerical illustration

Using Equations (4d), (7e) and (16d), we conduct a mathematical simulation to enumerate the breakeven level of the interest-free payday loan in each of the three models. The circuit exogenous factors encompass: (i) member income profile y; (ii) cost of fund prevailing in mainstream credit market r; (iii) transaction costs and default ζ and α; (iv) loan tenure and drawdown period n and t; (v) underwriting constraint corresponding to the income multiple b_1 and b_2; (vi) gestation period prior to loan drawdown m; (vii) fraction of borrowing members λ; and (viii) substantive circuit life T. We use the observations by Lawrence and Elliehausen (2008) and the FDIC (2010) to check the reasonableness of the exogenous parameters. Overall, the final values of the exogenous parameters are set to avoid excessive financial burden and ensure a liberal round of liquidity cycle, until member reaches financial security.

[Insert Table 1 here]

We tabulate the efficiency scenarios, given various permutations of the endogenous parameters, which cover (i) maximum member monthly contribution: C_{max}; (ii) maximum accumulated savings: S_{max}; and (iii) potential maximum loan: Q_{max}. Table 1 illustrates the effect on the endogenous factors, given changes in the exogenous parameters. This provides an indicative pricing framework that can be emulated in the design of similar endogenous leveraged circuits. It highlights the sensitivity of each endogenous factor to the decisions that the circuit undertakes and the various levers that may be combined to enhance the circuit efficiency.

Additionally, Table 2 provides the resultant values of the endogenous parameters, which assure that the circuit satisfies the Pareto-efficiency propositions under Models 1–3. Model 3 further demonstrates the interplay of the loan constraint covenant between $\frac{y}{b_2}$ or Q_{max}, where Q_{max} is characterised by the lower of either Q_{max1} or $Q_{max2,\ t\text{-}n}$. Here, $Q_{Binding}$ demands balancing the twin issues of: (i) protecting the member from potential debt entrapment; and (ii) ensuring the circuit's long-run liquidity, i.e., solvency. By not pursuing aggressive loan disbursement policies, it promotes accumulation of equity buffer that would ultimately allow the circuit greater financial latitude to pursue financial policies that enhance member welfare within the reasonable

risk tolerance limits, eg. relaxing the '$Q_{Binding}$' constraint and undertaking loan rehabilitation program that customises the loan repayment tenure for genuinely financially constrained member.

[Insert Table 2 here]

Based on Equation (16d), we extend the simulation to illustrate the effect of transaction cost on the potential loan amount, while holding other exogenous factors constant (see Figure 6).[34] Premised on the loan covenant in Equation (6) with loan multiple of $b_2 = 10$ and monthly after tax income of $y = \$2{,}000$, $\frac{y}{b_2}$ is then fixed at \$200. On the other hand, 'Q_{max}' changes with variation in the transaction cost 'ζ', i.e., there exists an inverse relationship between 'Q_{max}' and 'ζ'. '$Q_{Binding}$' as given in Equation (6a) is the minimum of either $\frac{y}{b_2}$ or Q_{max}. As highlighted in Equation (6b), if the transaction cost is low, i.e., $\frac{y}{b_2} < Q_{Max}$, then $Q_{Binding} = \frac{y}{b_2}$. Otherwise, $Q_{Binding}$ is restrained by Q_{max}. The critical transaction load, whereby $\frac{y}{b_2} = Q_{max}$, is when $\zeta = 28.7\%$, which is signified in Figure 6 by $\zeta_{critical}$.

[Insert Figure 6 here]

5. Discussion and conclusion

Payday borrowers are categorically those who suffer from poor credit history, exhibit time-inconsistent preferences and are often precluded by mainstream financiers. The prohibitive payday loan rates may potentially lead into a debt cycle if the borrower fails to observe the repayment term. Despite the unfavourable publicity against payday loans, financially constrained households still succumb to its services. This underlines a latent need for inexpensive short term liquidity facility to bridge their liquidity needs.

Unlike previous studies on payday loans, this study undertakes to conceptualise a solution to usurious payday loans and address credit rationing in mainstream credit. It entails the design of an institutional structure that embeds the interplay of cost efficient organisations and cultural beliefs. Our model is based on risk-neutral economic agents within an endogenous leverage circuit that draws from the technology of member based institutions such as ROSCA and its

34 Exogenous factors are: y = \$2,000, m = 6 months, r = 15%, α = 10%, λ = 0.4, T = 180 months, and t = 1.

hybrids, i.e., ASCRA, mutual and financial cooperative. A unique feature of this structure is that it harnesses the concept of coalition of savers and borrowers to allay inter-temporal liquidity shocks faced by its members, through the deployment of interest-free payday loan.

Our study illustrates the employment of this antiquated charitable form in contemporary financial perspective. This credit modality is chosen for its contrast with current payday loans. Furthermore, it is held esteemed in religious tenets, which specifically distinguish such deeds from usurious practices. We demonstrate that the circuit performs favourably in contrast to current usurious payday loans. First, the interest-free facility averts expropriation of wealth, an issue associated with payday loans. The periodic contribution, which features a minimal fraction of members' salary, promotes asset building which should consequently improve their financial security in the long-run. Second, our member based endogenous circuit allows credit accessibility to these households who are financially rationed by mainstream financiers. Third, we attest the economic proposition of interest-free loans expounded in religious teachings in current financial settings, whereby the circuit is able to boot strap its resources to grow endogenously. Fourth, in line with documented studies of time-inconsistent preferences of these households, the institutional design of our interest-free payday loan relies on the commitment technologies advocated with circuit-based structures.

The efficiency of an endogenous leveraged circuit is contingent on observing risk control measures to constraint adverse selection and moral hazard, thus reducing default and transaction costs significantly. This builds on ensuring equitable commitment, i.e., the periodic contribution can be fulfilled without jeopardising financial interests of individual members and the circuit. This is followed through by requiring the commitment to be directly dispensed into the circuit, which would effectively pre-empt irrational consumption tendencies. This is fortified with programs that build the members' financial capability and route them from poor credit tendencies. Next, we require existence of co-signer that acts in absence of standard collateral. Here, the co-signer's central role is in reducing costly state verification and execution of credible sanctions. The institutional design of the circuit provides upfront dilution of transaction costs that directly feeds into promoting the circuit efficiency.

Given the circuit technology that is member driven, accumulation of substantive capital base may create lag in time to loan issuance. To manage the gestation period, the circuit may rely on seed funding from *zakāt* and *sadaqah* to reinforce its initial capital base. These charitable funds can also be institutionalised to provide safety net to the circuit that defrays any long-run sustainability issues. Alternatively, the interest-free loan facility can be integrated into an already operational circuit, eg. financial cooperative. Results of our study support the policy

direction of the FDIC's (2010) small-dollar loan program. That is, it promotes affordable credit, observes risk-based underwriting, maximises technology and automation, integrates savings component in combination with financial education. Last, our model sets an indicative pricing mechanism, mostly absent in charitable institutions, which in the long-run promotes self-sufficiency.

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Appendix

A. Proof

We employ conditional probability premised on Bayes' rule with notations B: Borrow and NB: No Borrow, respectively to derive at:

(i) $$P\left(NB_{t+1-n}/NB_{t-n}\right)=\frac{P\left(NB_{t+1-n}\cap NB_{t-n}\right)}{P\left(NB_{t-n}\right)}$$

$$=\frac{P\left[NB_{t+1-n}\cap\left(\bigcup-B_{t-n}\right)\right]}{P\left(NB_{t-n}\right)}, \text{ where } \bigcup \text{ is the universal set}$$

$$=\frac{P\left[NB_{t+1-n}\cap\bigcup\right]-P\left[NB_{t+1-n}\cap B_{t-n}\right]}{P\left(NB_{t-n}\right)}$$

$$=\frac{P\left[NB_{t+1-n}\right]-P\left[NB_{t+1-n}\cap B_{t-n}\right]}{P\left(NB_{t-n}\right)}$$

$$=\frac{\left(1-\lambda_{t+1-n}\right)-\left(1-\rho_{t+1-n}\right)\lambda_{t-n}}{1-\lambda_{t-n}}$$

Subsequent reiterations yield the following in steady state:

$$=\frac{\left(1-\lambda\right)-\left(1-\rho\right)\lambda}{1-\lambda}$$

$$=\frac{1-2\lambda+\rho\lambda}{1-\lambda}$$

(ii) $$P\left(B_{t+1-n}/NB_{t-n}\right)=1-P\left(NB_{t+1-n}/NB_{t-n}\right)$$

Subsequent reiterations yield the following in the steady state:

$$=1-\left[\frac{1-2\lambda+\rho\lambda}{1-\lambda}\right]$$

$$=\frac{1-\lambda-\left(1-2\lambda+\rho\lambda\right)}{1-\lambda}$$

$$=\frac{\lambda\left(1-\rho\right)}{1-\lambda}$$

B. Figures

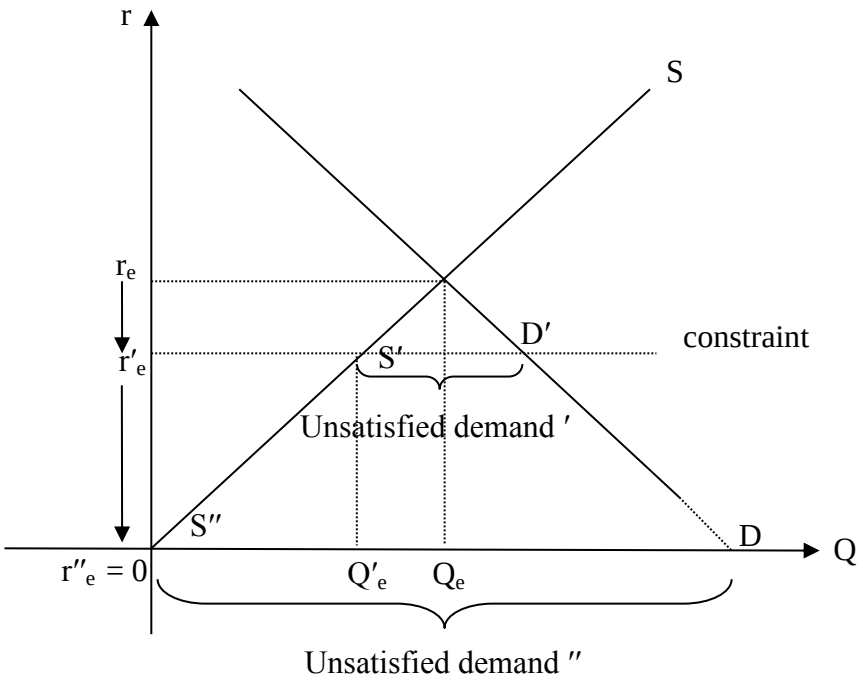

Figure 1. Supply and demand of funds with changes to the expected returns.

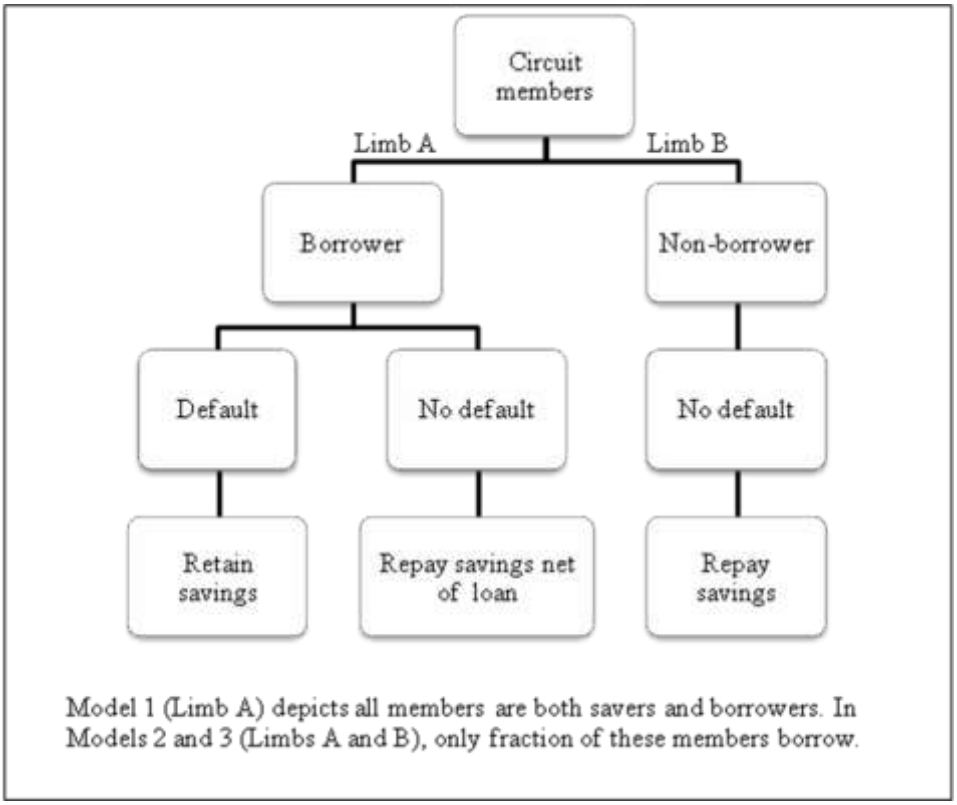

Figure 2. Stylised depiction of the various scenarios for Models 1-3.

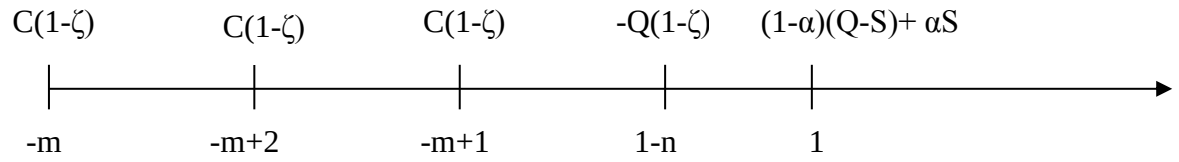

Figure 3. All members are savers and also liquidity constrained borrowers.

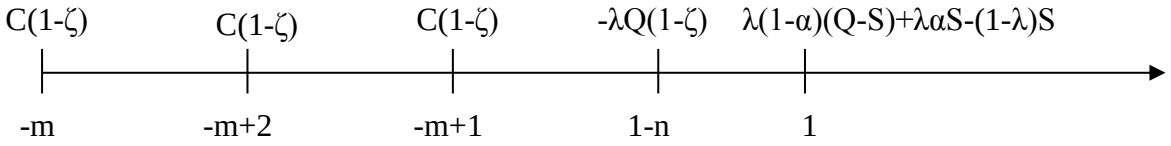

Figure 4. A fraction of liquidity constrained members borrow from the circuit 'λ' and supported by '1-λ' non-borrowing members.

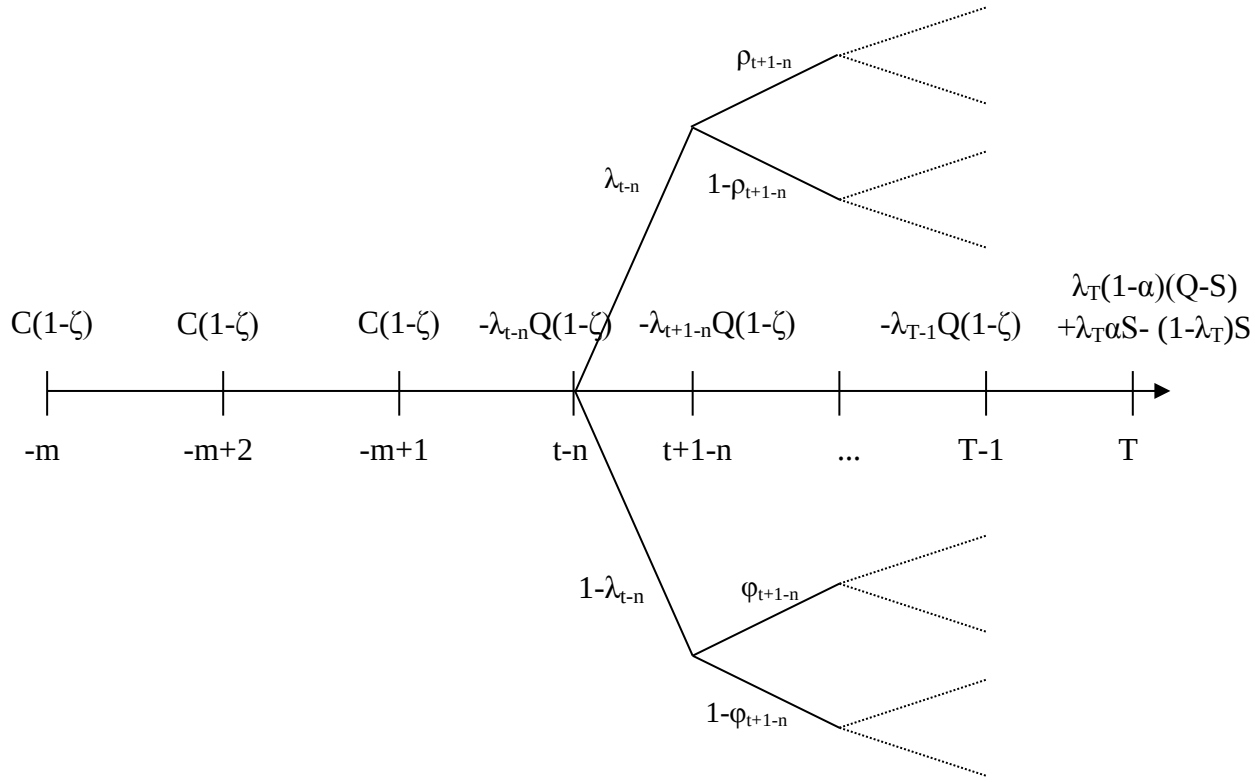

Figure 5. A fraction of liquidity constrained members borrow from the circuit 'λ' across multi-periods that in the long-run approaches a steady state.

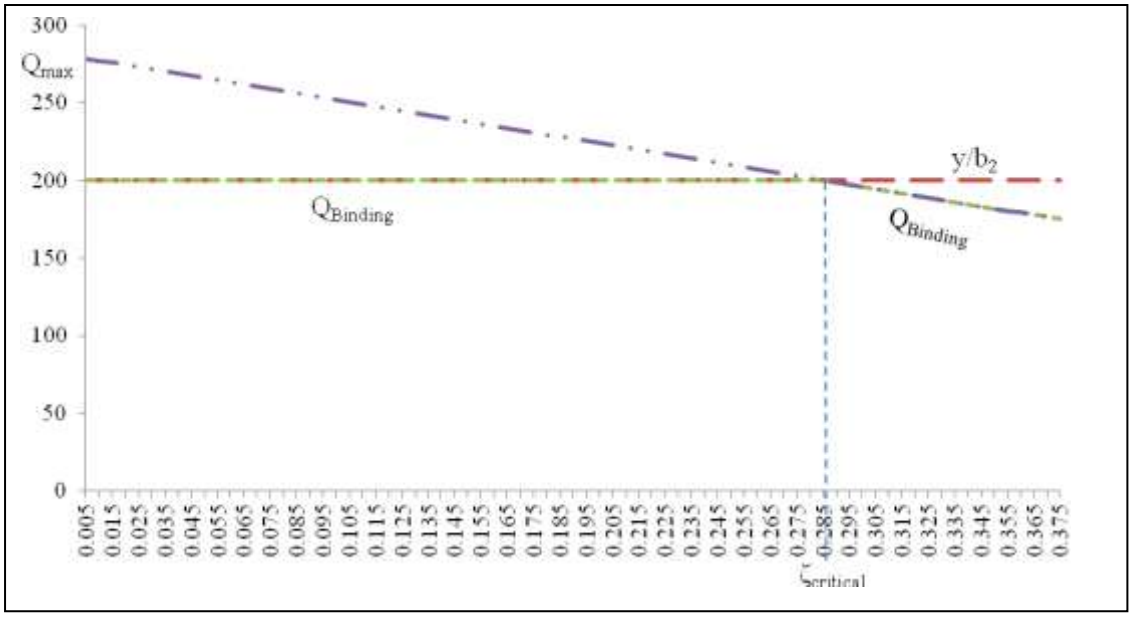

Figure 6. Effect of transaction cost on the circuit potential maximum loan.

C. Tables

Table 1. Indicative pricing structure of the endogenous interest-free payday loan circuit

Increase in exogenous factor	Direction of change in endogenous factor		
	C_{max}	S_{max}	Q_{max}
y	+	+	+
b_1	+	+	+
b_2	uc	uc	+
r	uc	uc	+
ζ	uc	uc	–
α	–	–	–
m	+	+	+
n	uc	uc	–
λ	uc	uc	–
T	uc	uc	+

Notes: Direction of change in the endogenous values '+' increase; '–'decrease, and 'uc' unchanged.

Table 2. Results illustrating circuit Pareto-efficiency for each model

Model 1										
b_1	b_2	r	ζ	α	m	n	C_{max}	S_{max}	Q_{max1}	$Q_{Binding}$
100	10	12%	2%	10%	6	¼ ⅓	20	160	1,045 960	200
		15%	2%	10%	6	¼ ⅓	20	160	1,228 1,113	200
50	10	12%	2%	10%	6	¼ ⅓	40	320	2,091 1,921	200
		15%	2%	10%	6	¼ ⅓	40	320	2,456 2,226	200

Model 2											
b_1	b_2	r	ζ	α	m	n	λ	C_{max}	S_{max}	Q_{max1}	$Q_{Binding}$
100	10	12%	2%	10%	6	¼	0.2 0.3 0.4	20	160	4,044 2,794 2,170	200
						⅓	0.2 0.3 0.4	20	160	3,715 2,567 1,993	200
		15%	2%	10%	6	¼	0.2 0.3 0.4	20	160	5,026 3,444 2,652	200
						⅓	0.2 0.3 0.4	20	160	4,555 3,121 2,404	200
50	10	12%	2%	10%	6	¼	0.2 0.3 0.4	40	320	8,087 5,589 4,339	200
						⅓	0.2 0.3 0.4	40	320	7,430 5,135 3,987	200
		15%	2%	10%	6	¼	0.2 0.3 0.4	40	320	10,052 6,887 5,305	200
						⅓	0.2 0.3 0.4	40	320	9,109 6,241 4,807	200

Model 3													
b_1	b_2	r	ζ	α	m	n	λ	T	C_{max}	S_{max}	Q_{max1}	Q_{max2}	$Q_{Binding}$
100	10	12%	2%	10%	6	¼	0.2 0.3 0.4	180	20	3,740	2,417 1,611 1,208	137	137
						⅓	0.2 0.3 0.4	180	20	3,740	2,394 1,596 1,197	137	137
						¼	0.2 0.3 0.4	240	20	4,940	2,417 1,611 1,208	137	137
						⅓	0.2 0.3 0.4	240	20	4,940	2,394 1,596 1,197	137	137
		15%	2%	10%	6	¼	0.2 0.3 0.4	180	20	3,740	2,279 1,520 1,140	137	137
						⅓	0.2 0.3 0.4	180	20	3,740	2,253 1,502 1,127	137	137
						¼	0.2 0.3 0.4	240	20	4,940	2,279 1,520 1,140	137	137
						⅓	0.2 0.3 0.4	240	20	4,940	2,253 1,502 1,127	137	137
50	10	12%	2%	10%	6	¼	0.2 0.3 0.4	180	40	7,480	4,833 3,222 2,417	274	200
						⅓	0.2 0.3 0.4	180	40	7,480	4,788 3,192 2,394	274	200
						¼	0.2 0.3 0.4	240	40	9,880	4,833 3,222 2,417	274	200
						⅓	0.2 0.3 0.4	240	40	9,880	4,788 3,192 2,394	274	200
		15%	2%	10%	6	¼	0.2 0.3 0.4	180	40	7,480	4,559 3,039 2,279	274	200
						⅓	0.2 0.3 0.4	180	40	7,480	4,506 3,004 2,253	274	200
						¼	0.2 0.3 0.4	240	40	9,880	4,559 3,039 2,279	274	200
						⅓	0.2 0.3 0.4	240	40	9,880	4,506 3,004 2,253	274	200

Notes: The model is solved for endogenous variables C_{max}, S_{max} and Q_{max} where C_{max} is the maximum monthly contribution, S_{max} is the maximum savings accumulated from the contributions, and Q_{max} is the potential maximum loan per period. Total loan advanced is given by $Q_{Binding} = min\ \{y/b_2, Q_{max}\}$. The values of the endogenous variables depicted in the table above signify the breakeven threshold that ensures the circuit is Pareto-neutral. For this simulation, the exogenous parameters are: (i) member monthly after tax income: y = \$2,000; (ii) income multiplier constraint: $b_{1,1}$ = 80 times, $b_{1,2}$ = 40 times; (iii) loan multiplier constraint: b_2 = 25 times; (iv) cost of funds: r_1 = 12%, r_2 = 15%; (v) transaction and default costs: ζ = 2% and α =10%; (vi) membership gestation period: m = 6 months; (vii) fraction of borrowers in the circuit: λ_1 = 0.2, λ_2 = 0.3; λ_3 = 0.4; (viii) loan tenure: n_1 = 7 days (¼ month); n_2 = 10 days (⅓ month); (ix) loan commencement period: t = 1; and (x) circuit life: T_1 = 15 years (180 months), T_2 = 20 years (240 months).

Glossary of Arabic terms

Arabic term	Closest English meaning
fiqh	Islamic jurisprudence.
hiba	Voluntary gift.
ijtihād	Literally 'exertion'. It implies independent deduction of laws not self-evident from the primary sources, namely the *Qur'ān* and *Sunnah*.
qard	The act of extending interest-free loan from one's property without expectations of gains. It is also known as *qard hasan* or *qardah-yi hasanah*.
Qur'ān	The holy book of Islam.
ribā	An injunction protecting property rights. This is generally misinterpreted as usury *or* interest.
ribā an-nasi'ah	This is termed as evident *ribā*. It is generally an injunction to deter expropriation of assets on deferred exchanges. It also mitigates financial fragility and the exclusion of underprivileged from financial services.
sadaqah	Voluntary offering or alms from a person's wealth.
salaf	The granting of loan subjected to repayment at a later time.
shari'ah	Islamic law
sunnah	The practices of the Prophet Muhammad PBUH. Along with the *Qur'ān* and *Hadith* (recorded sayings of the Prophet Muhammad), it is a major source of *Shari'ah*, or Islamic law.
takāful	Islamic insurance that functions similar to mutuals.
waqf (awqāf – plural)	Philanthropic foundations involving the endowment of privately owned properties for charitable purposes in perpetuity.
zakāt	Literally 'cleansing or purity'. It implies a religious tax to be deducted from one's wealth to help the needy.