Archive of SID.ir

Iranian Journal of Fuzzy Systems

Volume 21, Number 2, (2024), pp. 161-179

Original paper

University of Sistan and Baluchestan

# A hybrid model for choosing the optimal stock portfolio under intuitionistic fuzzy sets

M. Rasoulzadeh[1], S. A. Edalatpanah[2], M. Fallah[3] and S. E. Najafi[4]

[1,3] *Department of Industrial Engineering, Central Tehran Branch, Islamic Azad University, Tehran, Iran*
[2] *Department of Applied Mathematics, Ayandegan Institute of Higher Education, Tonekabon, Iran*
[4] *Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran*

Mehrdad.ra@gmail.com, s.a.edalatpanah@aihe.ac.ir, mohammad.fallah43@yahoo.com, Najafi1515@yahoo.com

**Abstract**

In the dynamic world of financial investment, crafting an optimal stock portfolio that judiciously balances risk, return, and efficiency emerges as a critical challenge. Despite the wealth of research on financial portfolio optimization, prevailing methodologies predominantly emphasize either risk minimization or return maximization, often overlooking the imperative for a holistic strategy that simultaneously boosts efficiency and effectiveness. Addressing this gap in the literature, this study introduces an innovative four-objective model that intricately blends risk, return, and efficiency considerations for the strategic selection of stock portfolios.

This model ingeniously integrates the foundational principles of Markowitz's mean-variance analysis with the sophisticated network data envelopment analysis (NDEA) techniques, significantly refining the portfolio selection methodology. It further distinguishes itself by incorporating returns represented as trapezoidal intuitionistic fuzzy numbers, adeptly capturing the inherent uncertainties in financial returns. Additionally, the model employs the network data envelopment analysis's cross-efficiency principle, providing a nuanced measure of company performance. To effectively navigate the complexities of this model, we deploy the Non-dominated Sorting Genetic Algorithm II (NSGA-II) and a multi-objective genetic algorithm, demonstrating the model's capability to unearth optimal solutions efficiently. The comparative analysis highlights that the proposed model significantly outperforms the efficiency and effectiveness of existing models, marking a substantial advancement in portfolio optimization strategies.

*Keywords:* Portfolio optimization, Markowitz mean-variance model, network data envelopment analysis, cross-efficiency, intuitionistic fuzzy sets.

## 1 Introduction

Investing in the stock market offers potential avenues for wealth generation, but it also comes with inherent uncertainties and challenges. One of the major challenges faced by investors is the task of identifying an optimal stock portfolio that can optimize returns while effectively managing risk [36]. Portfolio optimization has been extensively studied in the field of finance, with researchers and professionals striving to develop robust methodologies to guide investment decisions. Choosing the optimal stock portfolio offers several key advantages for investors [34]. Firstly, an optimal portfolio aims to maximize returns while effectively managing risk. By carefully selecting a combination of stocks with varying risk profiles and potential returns, investors can strike a balance that aligns with their risk tolerance and investment goals [14].

Considerable research has been conducted regarding the selection of an optimal stock portfolio. Ramen et al. [36] proposed a suggested method for choosing the best portfolio of stocks using variable length NSGA-II. Nourahmadi

Corresponding Author: S. A. Edalatpanah
Received: March 2023; Revised: March 2024; Accepted: May 2024.
https://doi.org/10.22111/IJFS.2024.45118.7968

Archive of SID.ir

Archive of SID.ir



and Sadeqi [34] proposed a Machine Learning-Based Hierarchical Risk Parity Approach in a Case Study of a Portfolio Consisting of Stocks of the Top 30 Companies in Tehran Stock.

Fuzzy mean–semi variance and Data Envelopment Analysis (DEA) cross efficiency were incorporated in a fuzzy multi-objective portfolio selection model by Chen et al. [13]. Mehlawat and Gupta [31] incorporate short-term returns, long-term returns, and liquidity in the portfolio in the portfolio optimization problem. Mehlawat et al. [32] proposed a multi-objective portfolio selection problem where the objectives included mean, variance, skewness, kurtosis, and efficiency. Rezaei Pouya et al. [39] solve a multi-objective portfolio optimization problem using invasive weed optimization based on fifty top companies of the Tehran Stock Exchange. Wang et al. [46] use the Sharpe ratio and Value-at-Risk ratio in fuzzy environments for portfolio optimization.

But the previous articles did not provide a model to reduce variance and increase efficiency and effectiveness at the same time. For this reason, in the present study, in order to reduce the variance and increase the efficiency and effectiveness of the stock portfolio, we present a four-objective model at the same time.

The present study introduces a novel model based on risk, return, and efficiency for choosing an appropriate stock portfolio. The model integrates the principles of Markowitz mean-variance analysis and network data envelopment analysis to enhance the portfolio selection process. By considering multiple objectives, including maximizing average stock return and effectiveness while minimizing associated risks, the model aims to provide a comprehensive approach to portfolio optimization. To address the uncertainty associated with the results, the study employs the concept of trapezoidal intuitionistic fuzzy numbers to represent returns. This allows for the representation of uncertainty and provides a more comprehensive understanding of the potential outcomes of the recommended portfolio. In various intuitionistic fuzzy sets, this investigation suggested a novel ranking function taking into account the communication of the membership functions with non-membership types.

The effectiveness of each decision-making unit can be better understood with the help of network data envelopment analysis because it makes it possible to look inside each unit's internal workings. The NDEA cross-efficiency model is employed in this research to analyze the firm's effectiveness [19]. To illustrate the uncertainties in returns, we take into account intuitionistic fuzzy returns. We also present a new four-objective model that incorporates the NDEA cross-efficiency into the Markowitz model with intuitionistic fuzzy efficiencies. To solve the model, the recommended new approach employs the NSGAII.

Furthermore, the paper incorporates the principle of network data envelopment analysis cross-efficiency to effectively measure company efficiency. By considering the efficiency of individual companies within the portfolio selection process, the model aims to enhance the overall performance of the chosen portfolio. To solve the complex optimization problem, the Non-Dominated Sorting Genetic Algorithm (NSGA-II) and multi-objective genetic algorithm are employed. These algorithms offer efficient and effective solutions by exploring the trade-off between multiple objectives and providing a set of optimal solutions.

The most popular mathematical model for assessing the performance of businesses is the data envelopment analysis (DEA) model presented by Charnes and Cooper in 1978. One of the useful tools in performance measurement that can offer effective units for portfolio formation is the cross-efficiency model using Data Envelopment Analysis. Traditional DEA models, nevertheless, take a black-box approach and overlook the internal organization of the units. In order to assess systems with more than one stage, Network Data Envelopment Analysis (NDEA) was developed [45].

The Markowitz model is combined with the network data envelopment analysis cross-efficiency moded trapezoidal intuitionistic fuzzy returns in this article. The financial structure of this hybrid model is determined by taking into account the market values of companies. In order to reduce variance and increase the return and effectiveness of the stock portfolio, we concurrently present a four-objective model.

The rest of the paper is organized as follows: A Literature Review is presented in Section 2. Details about the proposed method are presented in Section 3 Numerical Experiments are presented in Section 4. Results and Model parameters and Input and output values are presented in Section 5. Section 6 concludes the paper.

## 2 Literature review

Transaction costs were included in the constraints introduced by Yoshimoto [49], who also used a multi-period model. The number of asset constraints in the Markowitz model portfolio was also counted by Chang et al. [9] in 2000. The

Archive of SID.ir

Archive of SID.ir



upper and lower constraints of each portfolio asset were taken into account by Gómez and Sharma [22], who then introduced the Markowitz mean-variance model with limited components. Due to the fact that risk is described by variance in the Markowitz model, this criterion quantifies the excess or deficiency of return relative to investment risk. A new definition of risk has also been proposed by several investigators. Other than Markowitz's model, different definitions of risk have been employed, including half-variance and mean absolute deviation, by Chang et al. [11].

Consideration of a fixed return on assets can be less compliant with actual problems because there are constantly a variety of uncertain factors affecting the return on assets. Fuzzy returns can be taken into account as one of the best models for asset returns since the Markowitz model was developed. Other researchers have defined other types of return uncertainty and presented other models. Watada [48], and León et al. [28] investigated the problem of stock portfolio selection by employing the fuzzy decision theory. Interval planning models for supply chain design and portfolio selection were developed by Wang and Zhu [47], and Giove et al. [21]. Carlsson et al. [7] suggested a new method for selecting the most favorable portfolio based on the assumption that asset returns are trapezoidal fuzzy numbers. A fuzzy combined programming model was developed by Carlsson et al. [6] for selecting the best portfolio by taking prospective cash flows represented by trapezoidal fuzzy numbers into account.

To unravel the Markowitz mean-variance model and create a fuzzy portfolio selection model, Guo et al. [24] employed the fuzzy numbers concept. Edirisinghe and Zhang [18] suggested utilizing financial proportions to investigate company performance and the data envelopment analysis technique to determine a relative financial strength index (RFS), a criterion of a company's competitiveness relative to other corporates. Network models can address the problems because traditional data envelopment analysis models perform like black boxes. When introducing the network range-adjusted measure (NRAM) model in 2012, Avkiran and McCrystal [3] also compared the proposed model to the network slacks-based measure (NSBM) model and examined its sensitivity. The impact of the CNG2020 strategy on airline environmental performance was examined by Li [29] using the NRAM.

Chang et al. [10] assessed the portfolio using NDEA. The Markowitz and data envelopment analysis models for assessing the effectiveness of businesses do not take into account the concepts of risk and return. In other words, the effectiveness of companies in selecting the stock portfolio is not taken into account in the developed Markowitz models; this problem can be addressed by integrating Markowitz models with data envelopment analysis. Non-dominated sorting genetic algorithm II (NSGAII) was used by Omrani and Mashayekhi [35] in their presentation of a multi-objective model for portfolio selection. In addition to assessing the company's performance using financial ratio indicators, this model concurrently examines risk and return using the Markowitz model. Additionally, they measured company performance using the idea of cross-efficiency.

The 52 active companies from the Tehran Stock Exchange market make up the executed model. This model was compared to the NSGAII algorithm by Mashayekhi and Omrani [30], who viewed the return on assets as fuzzy trapezoidal numbers. The findings demonstrate that the returns, in this case, were substantially improved. Intuitionistic fuzzy data is one type of vague data that has been taken into account by some researchers and discussed in several investigations. Utilizing intuitionistic fuzzy input and output data, Puri and Yadav [40] conducted a groundbreaking analysis of optimistic and pessimistic efficiency. A triangular intuitionistic fuzzy environment was employed to introduce an extended DEA model by Edalatpanah [17]. Rasoulzadeh et al. [41] presented a four-objective model by combining Markowitz and DEA cross-efficiency models with intuitionistic fuzzy returns. A summary of the conducted research and research discussion is given in Table 1.

Archive of SID.ir

Archive of SID.ir



Table 1: Research literature review

| Ref | Purpose | Intuitionistic Fuzzy Set | DEA | NDEA | Markowitz models | Method | The number of decision units | Case study | Uncertainty |
|---|---|---|---|---|---|---|---|---|---|
| García-Melón et al [20] | prioritize project portfolio | No | No | No | No | RAI | 14 | The electrical sector of Venezuela | No |
| Bjerring et al [5] | Mitigates the problem of excessive portfolio | No | No | No | Yes | Out-of-sample Back-tests | 49 | Kenneth French data library | Yes |
| Peralta and Zareei [38] | Improve the portfolio selection process | No | No | No | Yes | In-sample and out-of-sample analysis | 200 | NYSE | No |
| Huang and Di [25] | Optimal portfolio selection | No | No | No | Yes | Multi-Greedy heuristic | 20 | Shanghai Stock Exchange | No |
| Rather et al [42] | Finding similar time windows and predicting the market behavior | No | No | No | Yes | KMNLOG KMDLOG SPCLOG and HRCLOG | 30 | — | NO |
| Pedersen and Peskir [37] | Compare the optimal portfolio with background risk with that without background risk | No | No | No | No | Uncertainty theory | 20 | Shanghai Stock Exchange | Yes |
| Zhou and Xu [51] | Forming an optimal portfolio of stocks | Yes | No | No | Yes | AI | 20 | Stock Exchange | No |
| Zhou Xiaoyang et al [50] | Stock portfolio selection | Yes | Yes | No | Yes | DEA | 33 | Chinese stock market | NO |
| Cesarone et al [8] | Prediction of stocks | No | No | No | Yes | Monte Carlo method Generating Gaussian random noise | 7 | Shanghai stock | Yes |
| Salehpoor et al [43] | Reduce estimation error and highly concentrated optimal portfolios | No | No | No | Yes | Markov-Switching (M-S) | 30 | US stocks | No |
| Khedmati and Azin [26] | the nonlinear mean-variance optimal control | No | No | No | Yes | Lagrange multipliers | 20 | — | No |
| Shadabfar and Cheng [44] | Choosing the Optimal Stock Portfolio | Yes | No | Yes | Yes | Electromagnetism-like algorithm (EM), particle swarm optimization (PSO), genetic algorithm (GA), genetic network programming (GNP) and simulated annealing (SA) | 50 | Iranian stock exchange | No |
| Chen et al [13] | Portfolio selection based on qualitative information | Yes | No | No | No | HFS | 3 | Stock market in China | Yes |
| Amin and Hajjami [1] | The nonlinear mean-variance optimal control | Yes | No | No | No | VaR $\epsilon$-constraint method | 10 | Chinese stock market | No |
| Oscar et al [16] | Solve the portfolio selection problem | No | No | No | No | multi-objective optimization Genetic algorithm | 50 | Shanghai Stock Exchange | Yes |
| Current | Choosing the Optimal Stock Portfolio | Yes | Yes | Yes | Yes | NSGAII and intuitionistic fuzzy efficiencies | 50 | Tehran Stock Exchange | Yes |

Archive of SID.ir

Archive of SID.ir



### 2.1 Contribution of the present study

In various intuitionistic fuzzy sets, this investigation suggested a novel ranking function taking into account the communication of the membership functions with non-membership types. The effectiveness of each decision-making unit can be better understood with the help of network data envelopment analysis because it makes it possible to look inside each unit's internal workings. The NDEA cross-efficiency model [33] is employed in this research to analyze the firm's effectiveness. To illustrate the uncertainties in returns, we take into account intuitionistic fuzzy returns. We also present a new four-objective model that incorporates the NDEA cross-efficiency into the Markowitz model with intuitionistic fuzzy efficiencies. To solve the model, the recommended new approach employs the NSGAII.

## 3 Methodology

In this section, the research methodology is addressed.

### 3.1 Markowitz model

Many improvements and refinements have been made to the original Markowitz model since it was first introduced because of its widespread acceptance as a useful resource for investors seeking portfolio optimization. The following describes the two-objective Markowitz model, in which the objective functions are maximizing returns and reducing risk:

$$Max \sum_{i=1}^{N} W_i \bar{R}_i,$$

$$Min\ \sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} W_i W_j cov\left(R_i, R_j\right),$$

s.t.

$$\sum_{i=1}^{N} W_i = 1,$$

$$W_i \geqslant 0 (i = 1, 2, 3, \cdots, N).$$

Where $\bar{R}_i$ denotes the mean return on assets $i$, $\mathrm{cov}(R_i, R_j)$ represents the return covariance on assets $i$ and $j$, $N$ represents the frequency of investable assets, and $w_i$ represents the weight of asset $i^{th}$ in the investment portfolio.

### 3.2 Intuitionistic fuzzy numbers

A trapezoidal intuitionistic fuzzy number $\widetilde{A}^I$ can be regarded as $A = (a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4)$ in which membership functions $\mu_{\widetilde{A}^I}$ and non-membership $\nu_{\widetilde{A}^I}$ are expressed as follows [40]:

$$\mu_{\tilde{A}^I} = \begin{cases} f_A(x) & a_1 \leq x < a_2 \\ 1 & a_2 \leq x \leq a_3 \\ g_A(x) & a_3 < x \leq a_4 \\ 0 & otherwise \end{cases}$$

$$\nu_{\tilde{A}^I} = \begin{cases} h_A(x) & b_1 \leq x < b_2 \\ 0 & b_2 \leq x \leq b_3 \\ k_A(x) & b_3 < x \leq b_4 \\ 1 & otherwise \end{cases}$$

Where $0 \leq \mu_{\widetilde{A}^I}(x) + \nu_{\widetilde{A}^I}(x) \leq 1$ and $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in \mathbb{R}$ and $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$. The functions $f_A$ and $k_A$ represent the non-descending continuous functions in the intervals $[a_1, a_2)$ and $(b_3, b_4]$ respectively, and the functions $g_A$ and $h_A$ are the non-ascending continuous functions in the intervals $(a_3, a_4]$ and $[b_1, b_2)$ respectively. The anticipated distance of an Intuitionistic phase number $\widetilde{A}^I$ defined as below [23]:

$$E_L\left(\tilde{A}^I\right) = \frac{b_1 + a_2}{2} + \frac{1}{2}\int_{b_1}^{b_2} h_A(x)\,dx - \frac{1}{2}\int_{a_1}^{a^2} f_A(x)\,dx,$$

Archive of SID.ir

Archive of SID.ir



$$E_U\left(\tilde{A}^I\right) = \frac{a_3+b_4}{2} + \frac{1}{2}\int_{a_3}^{a_4} g_A(x)\,dx - \frac{1}{2}\int_{b_3}^{b_4} k_A(x)\,dx.$$

Where $E_L$ is the lower limit of the expected value of intuitionistic fuzzy, $E_U$ is the upper limit of the expected value of intuitionistic fuzzy, and $E$ is the global expected value.

As a result, the center of the expected interval of that intuitionistic fuzzy number is determined as follows:

$$EV\left(\tilde{A}^I\right) = \frac{E_L\left(\tilde{A}^I\right) + E_U\left(\tilde{A}^I\right)}{2},$$

by hypothesis

$$f_A(x) = \frac{x-a_1}{a_2-a_1},$$

$$g_A(x) = \frac{x-a_4}{a_3-a_4},$$

$$h_A(x) = \frac{x-b_2}{b_1-b_2},$$

$$k_A(x) = \frac{x-b_3}{b_4-b_3},$$

In this case, we have [23]:

$$EV(A) = \frac{1}{8}\left(\sum_{i=1}^{4} a_i + \sum_{i=1}^{4} b_i\right),$$

To calculate the variance, we have [23]:

$$VAR(X) = E\left(X^2\right) - \left(E(X)\right)^2,$$

$$E_L\left(X^2\right) = \frac{b_1+a_2}{2} + \frac{1}{2}\int_{b_1}^{b_2} h_A\left(X^2\right)dx - \frac{1}{2}\int_{a_1}^{a_2} f_A\left(X^2\right)dx,$$

$$E_U\left(X^2\right) = \frac{a_3+b_4}{2} + \frac{1}{2}\int_{a_3}^{a_4} g_A\left(X^2\right)dx - \frac{1}{2}\int_{b_3}^{b_4} k_A\left(X^2\right)dx,$$

$$E_L\left(X^2\right) = -\frac{1}{3}\left(a_1^2+a_2^2+b_1^2+b_2^2+a_1a_2+b_1b_2\right) + a_1+a_2+b_1+b_2,$$

$$E_U\left(X^2\right) = -\frac{1}{3}\left(a_3^2+a_4^2+b_3^2+b_4^2+a_3a_4+b_3b_4\right) + a_3+a_4+b_3+b_4,$$

$$E\left(X^2\right) = \frac{E_L\left(X^2\right) + E_U\left(X^2\right)}{2}.$$

By placing relations (18) and (19) in relation (20), the value of $E(x^2)$ is obtained, and then with the help of relations (14) and (15), the variance of a trapezoidal intuitionistic fuzzy number $\widetilde{A}^I = (a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4)$ can be calculated as follows [23].

$$VAR\left(\tilde{A}^I\right) = \frac{1}{4}\left(\sum_{i=1}^{4} a_i + \sum_{i=1}^{4} b_i\right) - \frac{1}{12}\left(a_1a_2 + a_3a_4 + b_1b_2 + b_3b_4 + \sum_{i=1}^{4} a_i^2 + \sum_{i=1}^{4} b_i^2\right) - \frac{1}{64}\left(\sum_{i=1}^{4} a_i + \sum_{i=1}^{4} b_i\right)^2.$$

Archive of SID.ir

Archive of SID.ir



### 3.3 Data envelopment analysis cross-efficiency model

The DEA model compares the effectiveness of many DMUs that make use of various inputs to generate various outputs. Many scholars have built upon the original DEA model given by Charnes, Cooper, and Rhodes [12]. We assume that there are $n$ DMUs with $m$ inputs and $s$ outputs. The following model will find the set of weights that maximize the efficiency score of DMU $o$ where $x_{io}$ and $y_{ro}$ represent the inputs and outputs of the decision unit $o$:

$$Max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} \nu_i x_{io}},$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} \nu_i x_{ij}} \leq 1,$$

$$u_r, \nu_j \geq 0 (r = 1, \cdots, s)(i = 1, \cdots, m)$$

Where $x_{ij}$ and $y_{rj}$ represent the inputs and outputs of the decision unit $j$; $u_r$ and $v_i$ represent the inputs and outputs' weights, respectively. Maximizing the ratio of the weighted sum of the outputs to the weighted sum of the inputs is entailed in the objective function as much as possible [3].

Other DMU units' cross-efficiency assessed by the unit $K$ can be determined using the relationship shown below if $u_r^*$ and $v_i^*$ are the best answers from the aforementioned model for the unit $K$.

$$e_{kl} = \frac{\sum_{r=1}^{s} u_r^* y_{rl}}{\sum_{i=1}^{m} \nu_r^* x_{il}}.$$

If the above method is repeated for other DMUs, we can form an $n \times n$ matrix of $e_{ij}$ where $i, j = 1, \ldots, n$. The crossover efficiency matrix, which is the resulting matrix, is indicated by the symbol $E$. When calculating the average for each column in the performance matrix above, such as column $l$, we refer to the result as a cross-efficiency score [27].

$$\bar{e}_l = \frac{1}{n} \sum_{k=1}^{n} e_{kl}.$$

### 3.4 Cross-performance network data envelopment analysis model

Network DEA (Data Envelopment Analysis) is an extension of the traditional DEA methodology that allows for the evaluation of the relative efficiency and performance of interconnected units or entities within a network or system. It is particularly useful in assessing the efficiency and effectiveness of entities that depend on each other and operate collectively as a network [4].

In a network DEA analysis, the entities or decision-making units (DMUs) are interconnected through input-output relationships. The performance of each DMU is evaluated by considering both its individual efficiency and its contribution to the overall network efficiency. This approach enables the assessment of the network as a whole and identifies the synergies and interdependencies among the DMUs. The network DEA model takes into account the inputs and outputs of each DMU, as well as the network structure and the relationships among the entities. It aims to identify the optimal resource allocation and operational strategies that can improve the efficiency and overall performance of the network [3, 10, 30].

The network DEA analysis provides insights into various aspects of the network, such as the efficiency of individual DMUs, the efficiency of the overall network, and the potential for improving the network efficiency through changes in resource allocation or operational practices. It helps in identifying the best-performing entities within the network and understanding the factors that contribute to their success. By applying network DEA, decision-makers can identify inefficiencies, allocate resources more effectively, and enhance the overall performance of interconnected entities within a network [10, 30].

Overall, network DEA is a powerful analytical tool for evaluating the efficiency and performance of interconnected entities in a network setting. It provides a comprehensive understanding of the network's dynamics and enables informed decision-making for optimizing resource allocation and improving the overall efficiency of the network. In the present study, we employed network DEA because our study involved analyzing interconnected entities within a network. We wanted to assess the efficiency and performance of these entities while considering their interdependencies and the overall network dynamics. By using network DEA, we were able to capture the synergies, optimize resource allocation,

Archive of SID.ir

Archive of SID.ir



and obtain a more comprehensive evaluation of the network's performance, which would not have been possible with classical DEA [30].

Network DEA offers several advantages over classical data envelopment analysis (DEA) in the context of evaluating interconnected entities within a network. Here are some of the advantages and the reasons for using this approach in our research article:

1. Consideration of Interdependencies: Network DEA takes into account the interdependencies and relationships among entities within the network. This enables a more realistic and comprehensive evaluation of efficiency and performance, considering the collective impact of the entities on the overall network. In contrast, classical DEA analyzes entities in isolation and does not consider the interconnected nature of the network.

2. Synergy Analysis: Network DEA allows for the identification and quantification of synergy effects within the network. It captures the potential benefits and efficiencies that can be achieved by entities working together and leveraging their interdependencies. Classical DEA lacks the ability to evaluate and measure these synergy effects.

3. Resource Allocation Optimization: By assessing the efficiency and performance of interconnected entities, network DEA helps optimize resource allocation within the network. It provides insights into the most effective allocation of resources among entities to enhance the overall efficiency and performance of the network. This aspect is not addressed in classical DEA, which focuses on individual efficiency without considering resource allocation across the network.

4. Comprehensive Performance Evaluation: Network DEA provides a more holistic evaluation of the network by considering both individual entity efficiency and the overall network efficiency. This comprehensive approach allows for a deeper understanding of the network's performance, strengths, and areas for improvement. Classical DEA, on the other hand, only focuses on individual efficiency without capturing the broader network perspective.

NDEA model developed by Färe and Grosskopf [19] is employed to completely review the black box information. Classical Radial DEA (CRS) models should not be used because some inputs and outputs in financial models may have negative values. As a result, Classical Radial DEA models can be employed when some inputs or outputs are negative [15]. The Range Adjusted Measure (RAM) model with incompetence criteria will be as follows, given the preceding information:

$$Min - \frac{1}{m+s}\left(\sum_{i=1}^{m}\frac{s_{\bar{i}o}}{R_{\bar{i}}} + \sum_{r=1}^{s}\frac{s_{ro}^{+}}{R_{r}^{+}}\right),$$

s.t.

$$x_{io} = \sum_{j=1}^{n}\lambda_j x_{ij} + s_{\bar{i}o}(i = 1, \cdots, m),$$

$$y_{ro} = \sum_{j=1}^{n}\lambda_j y_{rj} - s_{ro}^{+}(r = 1, \cdots, s),$$

$$\sum_{j=1}^{n}\lambda_j = 1,$$

$$\lambda_j, s_{io}^{-}, s_{ro}^{+} \geq 0(j = 1, \cdots, n).$$

$X = x_{ij} \in \mathbb{R}^{m\times n}$ and $Y = y_{rj} \in \mathbb{R}^{s\times n}$ indicate the data matrix of input and output in which all columns demonstrate one of the units, and all rows show the level of one of the factors of the relative DMU. $s_{io}^{-}$ and $s_{ro}^{+}$ stand for the slacks of the $i$th input and $r$th output.

Also, $R^{-}$ and $R^{+}$ are expressed as follows:

$$R^{-} = \left(\frac{1}{R_1^{-}}, \frac{1}{R_2^{-}}, \frac{1}{R_3^{-}}, \cdots, \frac{1}{R_m^{-}}\right),$$

$$R^{+} = \left(\frac{1}{R_1^{+}}, \frac{1}{R_2^{+}}, \frac{1}{R_3^{+}}, \cdots, \frac{1}{R_s^{+}}\right),$$

Archive of SID.ir

Archive of SID.ir



$$R_i^- = {}^{max\{x_{ij}\}}_{j=1,\cdots,n} - {}^{min\{x_{ij}\}}_{j=1,\cdots,n},$$

$$R_i^+ = {}^{max\{y_{rj}\}}_{j=1,\cdots,n} - {}^{min\{y_{rj}\}}_{j=1,\ldots,n}.$$

The Figure 1 provides an overview of a network model for DMU number $l$.

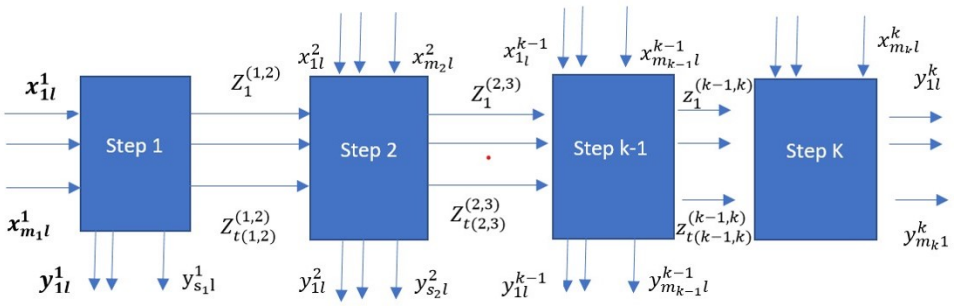


Figure 1: Overview of a network model for DMU number $l$

Where:
$x_{ij}^k$: is the $i$th input for the $j$th unit in the $k$th step, where $k = 1, \ldots, K$, $i = 1, \ldots, m_k$, and $j = 1, \ldots, n$.

$y_{rj}^k$: is the $r$th output for the $j$th unit in the $k$th step, where $k = 1, \ldots, K$, $r = 1, \ldots, s_k$, and $j = 1, \ldots, n$.

$Z_j^{(i-1,i)}$: $j$th output in the $i-1$th step and input of the $i$th step (internal link).

$t(i-1, i)$: output count of the $i-1$th stage and $i$th inputs (internal link).

The Min model with the performance criterion is as follows:

$$Min \sum_{k=1}^{K} - \frac{t_k}{m_k + s_k} \left( \sum_{i=1}^{m_k} \frac{s_i^{k-}}{R_i^-} + \sum_{r=1}^{m_k} \frac{s_r^{k+}}{R_r^+} \right).$$

s.t.

$$x_o^k = \sum_{j=1}^{n} \lambda_j^k x_{ij}^k + s_i^{k-} (i = 1, \cdots, m_k; k = 1, \cdots, K),$$

$$y_o^k = \sum_{j=1}^{n} \lambda_j^k y_{rj}^k - s_i^{k+} (r = 1, \cdots, s_k; k = 1, \cdots, K),$$

$$\sum_{j=1}^{n} \lambda_j^k Z_j^{(k.h)} = \sum_{j=1}^{n} \lambda_j^h Z_j^{(k.h)} (h, k = 1, \cdots, K),$$

$$\sum_{j=1}^{n} \lambda_j^k = 1 (k = 1, \cdots, K),$$

$$\lambda, s^-, s^+ \geq 0 (j = 1, \cdots, n),$$

where $\lambda^k \in \mathbb{R}^n$ is the intensity vector corresponding to Division $k$ $(k = 1, \ldots, K)$. $t_k$ is the weight of Division $k$, and $s_i^{(k-)}$ and $s_r^{(k+)}$ are the slacks of the $i$th input and $r$th output for Division $k$.
The following is the dual of the models mentioned above:

$$Max \sum_{k=1}^{K} \sum_{r=1}^{s_k} p_r^k y_o^k - \sum_{k=1}^{K} \sum_{i=1}^{m_k} q_i^k x_o^k + \sum_{k=1}^{K} \xi_k,$$

Archive of SID.ir

Archive of SID.ir



s.t.

$$\sum_{r=1}^{s_1} p_r^1 y_{rl}^1 - \sum_{i=1}^{m_1} q_i^1 x_{il}^1 + \sum_{j=1}^{t(1.2)} c_j^1 Z_j^{(1.2)} + \xi_1 \leq 0 (l = 1, \ldots, n),$$

$$\sum_{r=1}^{s_k} p_r^k y_{rl}^k - \sum_{i=1}^{m_k} q_i^k x_{il}^k - \sum_{j=1}^{t(k-1.k)} c_j^k Z_j^{(k-1.k)} + \xi_k \leq 0 (l = 1, \ldots, n),$$

$$\sum_{r=1}^{s_k} p_r^k y_{rl}^k - \sum_{i=1}^{m_k} q_i^k x_{il}^k - \sum_{j=1}^{t(k-1.k)} c_j^{k-1} Z_j^{(k-1.k)} + \sum_{j=1}^{t(k.k+1)} c_j^k Z_j^{(k.k+1)} + \xi_k \leq 0 (l = 1, \ldots, n; k = 2, ..., k-1),$$

$$p_i^j \geq \frac{w_j}{(m_j + s_j) R_i^+} (i = 1, \ldots, s_j; j = 1, \ldots, K),$$

$$q_i^j \geq \frac{w_j}{(m_j + s_j) R_i^-} (i = 1, \ldots, m_j; j = 1, \ldots, K).$$

Where **p**, **q**, and **c** vectors represent the price of output and the cost of input and units connection. $\xi_k$ is a positive infinitesimal value. The model maximizes DMU's efficiency score and optimizes the weight for all DMUs simultaneously. Let $^*$ represent the optimal solution of the model. The efficiency score of other DMUs is obtained by using the weights that DMU $k$ has chosen. The cross-efficiency of DMU $l$ with the weights of DMU $k$ ($e_{kl}$) can be expressed as follows [14]:

$$e_{kl}^* = p_k^* y_k - q_k^* x_k + \xi_k.$$

## 4 Markowitz model integrated with intuitionistic fuzzy efficiencies and network data envelopment analysis cross-efficiency

In order to concurrently evaluate risk, return, and efficiency, Omrani and Mashayekhi [35] presented a four-objective Markowitz model incorporated with the data envelopment analysis cross-efficiency model. Returns can be thought of in these models as either fuzzy or definite numbers. Despite the fact that fuzzy numbers are less precise and more uncertain than crisp ones, it may be assumed that a topic has a certain level of membership when there may be misgivings regarding that level. Take into account instances in which the variables in question are linguistic in nature, i.e., their values are not described numerically but rather through the use of words like "good," "bad," "short," "low," etc. There is no method available in fuzzy set theory to incorporate this ambiguity in membership levels.

The Intuitionistic Fuzzy Set (IFS) recommended by Atanassov [2] could be used to solve this problem. For a more realistic and precise assessment in this study, the returns are taken into consideration as intuitionistic fuzzy numbers. Additionally, the NDEA model has been employed to analyze performance because DEA models are comparable to the "black box" and have deficiencies in sub-process analysis. It means that these models are seen as complex and opaque. They take inputs and produce outputs, but the inner workings and mechanisms are not easily understandable or transparent.

This lack of transparency can make it difficult to gain insights into the specific sub-processes or components that contribute to overall performance. To overcome this limitation, the NDEA (Nested Data Envelopment Analysis) model has been employed. The NDEA model is a more advanced version of DEA that allows for a more detailed analysis of sub-processes. It provides a framework for breaking down the overall performance into smaller components and assessing the efficiency of each individual sub-process. The newly discovered model is as follows:

$$Max \qquad E\left(\sum_{i=1}^{N} \tilde{R}_i^I w_i\right) = \frac{1}{8}\left(\sum_{i=1}^{N} \left(a_{i1} + a_{i2} + a_{i3} + a_{i4} + b_{i1} + b_{i2} + b_{i3} + b_{i4}\right) w_i\right),$$

$$Min \qquad VAR\left(\sum_{i=1}^{N} \tilde{R}_i^I w_i\right),$$

Archive of SID.ir

Archive of SID.ir



$$= \frac{1}{4}\sum_{i=1}^{N}\left(\left(\sum_{j=1}^{4}a_{ij}+\sum_{j=1}^{4}b_{ij}\right)-\frac{1}{12}\left(a_{i1}a_{i2}+a_{i3}a_{i4}+b_{i1}b_{i2}+b_{i3}b_{i4}+\sum_{j=1}^{4}a_{ij}^{2}+\sum_{j=1}^{4}b_{ij}^{2}\right)-\frac{1}{64}\left(\sum_{j=1}^{4}a_{ij}+\sum_{j=1}^{4}b_{ij}\right)^{2}w_i\right),$$

$$Max \quad \sum_{i=1}^{N} w_i\tilde{e}_i,$$

$$Min \quad \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j cov\left(e_i, e_j\right),$$

s.t.

$$l_i z_i \leq w_i \leq u_i z_i (i = 1, ..., N),$$

$$\sum_{i=1}^{N} w_i = 1,$$

$$w_i \geq 0 (i = 1, \ldots, N).$$

Where the $z_i$ represents a zero-one variable. The value is 0 unless the asset $i$th is added to the portfolio, at which point it becomes 1. Maximum and minimum allocations of the portfolio's total investment budget to the $i$th variable are indicated by $l_i$ and $u_i$, respectively. The efficiency score of the network intersection of $i$th DMU is demonstrated by $\bar{e}_i$, enhancing portfolio effectiveness. The covariance of the network intersecting effectiveness between $e_{(.i)}$ (DMU$i$) and $e_{(.j)}$ (DMU$j$) is indicated by cov($e_i$ *and* $e_j$).

## 5 Numerical experiments

To solve a numerical expression and compare the answers, we use information about Tehran Stock Exchange companies. Fifty firms listed on the Tehran Stock Exchange in 2020 have been chosen for this purpose. These companies have not been added to the list of desired businesses because banks, insurance, and investment firms have distinct financial evaluations than others. Moreover, we employ the documented data in the financial declarations up to 2020 to assess the companies' effectiveness.

Performance has been more precisely calculated using a two-stage network model. Table 2 shows the inputs and outputs for the initial phase of the above model. While also increasing the firm's productivity and improving its parameters, the results of the second phase demonstrate the management's capacity to make profitable utilization of the company's resources. The effectiveness and results of the company's parameters are shown by these outputs. The market value of the company at the end of 2020, therefore, is the output of the second phase. Thus, the efficiency of the company and its effect on shareholder wealth can be better explained by this network. For shareholders and investors, this is the most important indicator of the company's productivity.

### 5.1 Input and output values

The efficiency in the initial phase was assessed using seven input and seven output criteria. Table 2 shows a collection of input and output criteria. Data has been extracted from and evaluated for each piece of information found in the audited financial statements up to 2020.

Archive of SID.ir

Archive of SID.ir



Table 2: Research Criteria

| Description | Criteria Type | input and output |
|---|---|---|
| Period income divided by accounts receivable | Turnover of accounts receivable | Input |
| Period income divided by inventories | Inventory turnover | Input |
| Revenue shared by wealth | Asset turnover | Input |
| Existing wealth over existing wealth | Current ratio | Input |
| Acidic assets on existing rabies debt | Quick ratio | Input |
| Overall debt shared by shareholders' justice | Debt to equity ratio | Input |
| The total debt on overall wealth | Debt ratio | Input |
| Net profit on equity | Return on equity (ROE) | Output |
| Net profit on assets | Return on assets (ROA) | Output |
| Net profit on sales | Net profit margin | Output |
| Net profit on the number of shares | Earnings per share (EPS) | Output |
| Existing period income divided by the earlier period income minus one | Income growth rate | Output |
| Current period net profit shared by previous period net profit minus one | Net profit growth rate | Output |
| Existing period EPS shared by earlier period EPS minus one | Earnings growth rate per share | Output |

Additionally, the Tehran Stock Exchange's website was used to obtain the second stage's output, which is the market value of businesses in 2020 as the last business day for the companies under evaluation. Table 3 shows the returns for the fifty corporates as intuitionistic fuzzy numbers. The MATLAB version 2014 software and NSGA-II algorithm were used to solve the presented model.

## 5.2 Model parameters

In this model, the population size $N_{\text{pop}}$ is considered one hundred, $p_c = 0.8$, $p_m = 0.1$, maximum number of repetitions $= 200$, $\mu = 2$; the minimum investment per share $(l_i)$ is considered 10% and the maximum investment per share $(u_i)$ is equal to 30%, and $t_1 = 0.7$ and $t_2 = 0.3$.

## 5.3 Results

The suggested model has been put into practice in this section, and we've got a list of Pareto-optimal answers as a result. These findings can be used to develop an investment portfolio for investors. The favored portfolio can be chosen by taking into account a variety of factors. The most efficient responses are taken into account in this model. Tables 4-7 demonstrate the acquired findings.

Archive of SID.ir

Archive of SID.ir



Table 3: Returns as trapezoidal intuitionistic fuzzy numbers

| **Asset No** | ***IF Returns*** | ***Asset No*** | ***IF Returns*** |
|---|---|---|---|
| 1 | *(0.01,0.14,0.262,0.39,-0.29,0.1,0.29,1.24)* | *26* | *(0.1,0.14,0.181,0.22,-0.25,0.13,0.19,0.69)* |
| 2 | (-0.05,0.05,0.152,0.25,-0.29,0.02,0.18,0.56) | 27 | (0.04,0.1,0.166,0.23,-0.23,0.08,0.18,0.55) |
| 3 | (0.01,0.07,0.122,0.18,-0.2,0.05,0.14,0.83) | 28 | (-0.06,0,0.063,0.12,-0.27,-0.01,0.08,0.67) |
| 4 | (-0.08,0,0.075,0.15,-0.4,-0.02,0.09,0.59) | 29 | (-0.02,0.04,0.097,0.16,-0.39,0.03,0.11,0.52) |
| 5 | (0.07,0.12,0.173,0.22,-0.22,0.11,0.19,0.73) | 30 | (0.07,0.1,0.127,0.16,-0.08,0.09,0.13,0.45) |
| 6 | (0.03,0.09,0.142,0.2,-0.25,0.07,0.16,0.56) | 31 | (-0.03,0.01,0.043,0.08,-0.42,0,0.05,0.25) |
| 7 | (-0.01,0.07,0.148,0.23,-0.31,0.05,0.17,0.81) | 32 | (-0.03,0.12,0.272,0.42,-0.3,0.08,0.31,1.42) |
| 8 | (0.04,0.07,0.095,0.12,-0.16,0.06,0.1,0.4) | 33 | (0.02,0.11,0.207,0.3,-0.17,0.09,0.23,1.01) |
| 9 | (-0.04,0.02,0.071,0.12,-0.3,0,0.08,0.54) | 34 | (0,0.03,0.05,0.07,-0.12,0.02,0.06,0.58) |
| 10 | (-0.01,0.02,0.063,0.1,-0.3,0.01,0.07,0.59) | 35 | (-0.05,0.03,0.122,0.21,-0.33,0.01,0.14,0.74) |
| 11 | (-0.02,0.03,0.085,0.14,-0.28,0.02,0.1,0.45) | 36 | (-0.06,0.02,0.098,0.18,-0.47,0,0.12,0.56) |
| 12 | (0.09,0.15,0.205,0.26,-0.2,0.14,0.22,0.63) | 37 | (-0.25,0.09,0.419,0.75,-0.47,0,0.5,1.77) |
| 13 | (-0.04,0.01,0.065,0.12,-0.28,0,0.08,0.51) | 38 | (-0.05,0.06,0.171,0.28,-0.27,0.03,0.2,1.17) |
| 14 | (0.09,0.14,0.184,0.23,-0.32,0.12,0.2,0.56) | 39 | (0.01,0.05,0.081,0.12,-0.21,0.04,0.09,0.45) |
| 15 | (-0.05,0.05,0.156,0.26,-0.28,0.03,0.18,0.51) | 40 | (0.08,0.16,0.241,0.32,-0.23,0.14,0.26,0.84) |
| 16 | (0.05,0.1,0.155,0.21,-0.12,0.09,0.17,0.51) | 41 | (-0.02,0.04,0.093,0.15,-0.31,0.02,0.11,0.71) |
| 17 | (-0.02,0.02,0.062,0.1,-0.23,0.01,0.07,0.51) | 42 | (0.01,0.04,0.063,0.09,-0.21,0.03,0.07,0.42) |
| 18 | (0.07,0.13,0.193,0.25,-0.24,0.12,0.21,0.9) | 43 | (0,0.03,0.062,0.09,-0.19,0.03,0.07,0.47) |
| 19 | (0,0.08,0.151,0.23,-0.24,0.06,0.17,0.79) | 44 | (0.05,0.08,0.116,0.15,-0.17,0.07,0.12,0.65) |
| 20 | (0.06,0.15,0.238,0.33,-0.36,0.13,0.26,0.88) | 45 | (-0.16,0.1,0.361,0.62,-0.45,0.03,0.43,1.72) |
| 21 | (0,0.07,0.145,0.22,-0.16,0.06,0.16,0.61) | 46 | (0.09,0.11,0.124,0.14,-0.07,0.1,0.13,0.4) |
| 22 | (-0.07,-0.01,0.053,0.12,-0.27,-0.03,0.07,0.44) | 47 | (0.02,0.08,0.148,0.21,-0.27,0.06,0.16,0.47) |
| 23 | (0.05,0.13,0.202,0.28,-0.33,0.11,0.22,0.84) | 48 | (0.01,0.06,0.1,0.14,-0.26,0.04,0.11,0.6) |
| 24 | (0,0.09,0.176,0.27,-0.37,0.06,0.2,0.99) | 49 | (-0.04,0.07,0.193,0.31,-0.33,0.04,0.22,0.83) |
| 25 | (0.01,0.14,0.262,0.39,-0.29,0.1,0.29,1.24) | 50 | (0.15,0.18,0.207,0.23,-0.18,0.17,0.21,0.59) |

Table 4: Assets' weights with the IFS Network DEA model

| **Asset No** | 1 | 31 | 36 | 44 |
|---|---|---|---|---|
| **Weights** | 0.21 | 0.23 | 0.24 | 0.32 |

Table 5: Assets' weights with the IFS DEA model

| **Asset No** | 31 | 36 | 44 | 50 |
|---|---|---|---|---|
| **Weights** | 0.21 | 0.35 | 0.25 | 0.19 |

Archive of SID.ir

Archive of SID.ir



Table 6: Assets' weights with fuzzy set Network DEA model

| **Asset No** | 22 | 36 | 37 | 42 | 49 |
|---|---|---|---|---|---|
| **Weights** | 0.20 | 0.17 | 0.19 | 0.25 | 0.19 |

Table 7: Assets' weights with fuzzy set DEA model

| **Asset No** | 1 | 31 | 32 | 36 | 39 |
|---|---|---|---|---|---|
| **Weights** | 0.20 | 0.14 | 0.21 | 0.21 | 0.24 |

Table 8 also shows the findings for the objective function of both models.

Table 8: Optimal values of the objective functions

| **Objective** | **Objective Type** | **Target Function** | **Intuitionistic Fuzzy & Network** | **Fuzzy & Network** | **Intuitionistic Fuzzy** | **Fuzzy** |
|---|---|---|---|---|---|---|
| z1 | Maximum | Expected Return Rate | 0.33 | 0.31 | 0.30 | 0.28 |
| z2 | Minimum | Return Rate-Based Portfolio Risk | 0.19 | 0.18 | 0.20 | 0.12 |
| z3 | Maximum | Portfolio Effectiveness | 0.66 | 0.55 | 0.51 | 0.53 |
| z4 | Minimum | Effectiveness-Based Portfolio Risk | 0.09 | 0.10 | 0.15 | 0.16 |

Based on the provided data in Table 8, here is a comparison and the corresponding result:
Objective: Maximum Expected Return Rate

- Intuitionistic Fuzzy & Network: 0.33
- Fuzzy & Network: 0.31
- Intuitionistic Fuzzy: 0.30
- Fuzzy: 0.28

The highest expected return rate is associated with the Intuitionistic Fuzzy objective, with a value of 0.33. The Fuzzy & Network objective follows closely with a value of 0.31, while the Intuitionistic Fuzzy and Fuzzy objectives have lower values of 0.30 and 0.28, respectively.
Objective: Minimum Return Rate-Based Portfolio Risk

- Intuitionistic Fuzzy & Network: 0.19
- Fuzzy & Network: 0.18
- Intuitionistic Fuzzy: 0.20
- Fuzzy: 0.12

The lowest return rate-based portfolio risk is observed with the Fuzzy objective, having a value of 0.12. The Fuzzy & Network and Intuitionistic Fuzzy & Network objectives have slightly higher values of 0.18 and 0.19, respectively. The Intuitionistic Fuzzy objective has the highest value of 0.20, indicating a relatively higher risk.
Objective: Maximum Portfolio Effectiveness

- Intuitionistic Fuzzy & Network: 0.66

Archive of SID.ir

Archive of SID.ir



- Fuzzy & Network: 0.55
- Intuitionistic Fuzzy: 0.51
- Fuzzy: 0.53

The maximum portfolio effectiveness is achieved with the Intuitionistic Fuzzy & Network objective, having a value of 0.66. The Fuzzy & Network and Fuzzy objectives have values of 0.55 and 0.53, respectively, indicating a slightly lower level of effectiveness. The Intuitionistic Fuzzy objective has the lowest value of 0.51, indicating a relatively lower level of effectiveness.

Objective: Minimum Effectiveness-Based Portfolio Risk

- Intuitionistic Fuzzy & Network: 0.09
- Fuzzy & Network: 0.10
- Intuitionistic Fuzzy: 0.15
- Fuzzy: 0.16

The lowest effectiveness-based portfolio risk is associated with the Intuitionistic Fuzzy & Network objective, having a value of 0.09. The Fuzzy & Network and Fuzzy objectives have slightly higher values of 0.10 and 0.16, respectively. The Intuitionistic Fuzzy objective has the highest value of 0.15, indicating a relatively higher risk.

The intuitionistic fuzzy model outperforms the fuzzy model in terms of rate of return and effectiveness, as demonstrated by the Table above. Additionally, the effectiveness risk has decreased notwithstanding the fact that the risk of portfolio returns has increased significantly. Furthermore, when compared to the non-network model, the findings have improved dramatically, notwithstanding the effectiveness enhancement and risk reduction. Eventually, we can conclude that more pertinent findings in terms of effectiveness and efficiency can be acquired by taking into account the intuitionistic fuzzy returns as well as the network structure. The Figure 2 determines the final result of the problem.

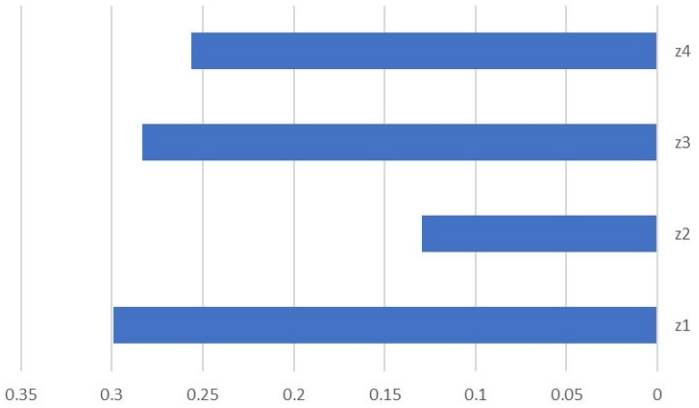


Figure 2: Final result

These weights represent the relative importance or priority assigned to each objective in the decision-making process. The target functions associated with each objective represent the specific metrics used to evaluate and measure the corresponding aspects. Based on the provided data, the objective with the highest weight is "Expected Return Rate" with a weight of 0.2992. This objective has the greatest relative importance in the decision-making process compared to the other objectives. On the other hand, the objective with the lowest weight is "Return Rate-Based Portfolio Risk" with a weight of 0.1297. This objective has the least relative importance among the given objectives. The weight assigned to each objective represents the relative importance or priority given to that objective in the decision-making process. The fact that the "Expected Return Rate" objective has a higher weight compared to the other objectives suggests that it is considered more important in the decision-making process.

There could be several reasons why the "Expected Return Rate" objective has a higher weight:

1. Investment goals: The decision-makers may have determined that maximizing the expected return rate is the primary goal of the investment strategy. This could be driven by factors such as the need for high profitability or meeting specific financial targets.

Archive of SID.ir

Archive of SID.ir



2. Risk appetite: The decision-makers may have a higher tolerance for risk and prioritize higher returns over other factors. They might believe that the potential for higher returns justifies taking on greater risk.

3. Strategic focus: The decision-makers may be placing a strategic emphasis on generating higher returns as a means of achieving competitive advantage or meeting specific organizational objectives.

4. Investor preferences: The weight assigned to the "Expected Return Rate" objective could also reflect the preferences and priorities of the investors or stakeholders involved in the decision-making process. They may have explicitly expressed a stronger preference for higher returns.

## 6 Conclusion

In this study, the intuitionistic fuzzy returns, network data envelopment analysis, and Markowitz models were integrated. Using financial statement data and company market value, the four-objective model takes into account the returns and risk, as well as the units' efficiency. The model was executed based on the price fluctuations over the course of three years from 2020 until 2023, as well as the financial data and market value of the most recent year for fifty publicly-traded companies. In the instances of fuzzy and intuitionistic fuzzy efficiencies, in addition to network and non-network DEA, the models were solved. The findings demonstrated that, when compared to the other models, the models' efficiency and effectiveness utilizing network intuitionistic fuzzy efficiencies had dramatically increased. We'll focus on various types of data uncertainty and the extended Markowitz model in upcoming investigations. Future research can explore the integration of various types of data uncertainty, such as imprecise and vague information, into the network intuitionistic fuzzy efficiencies model. Additionally, investigations should focus on extending the Markowitz model to incorporate additional factors that affect investment decisions, such as environmental, social, and governance (ESG) criteria. These advancements would enhance the accuracy and reliability of the decision-making process in portfolio management and provide more comprehensive insights for investors. Furthermore, future research should also consider the dynamic nature of financial markets and incorporate time-series analysis to capture the evolving trends and patterns in asset returns and risk. This could involve developing models that adapt to changing market conditions and incorporate real-time data updates. Additionally, exploring the application of machine learning and artificial intelligence techniques, such as deep learning and neural networks, can provide more robust and accurate predictions for portfolio management. Moreover, incorporating alternative data sources, such as social media sentiment analysis or satellite imagery, can offer valuable insights into market dynamics and help in identifying new investment opportunities. Finally, conducting empirical studies with a larger sample size and across different industries can validate the effectiveness and generalizability of the proposed models and strategies. For future research, it is strongly recommended to investigate the following:

1. Integration of Machine Learning Techniques: The researchers could explore the integration of machine learning techniques, such as Biogeography-based Optimization of Artificial Neural Network (BBO-ANN), into the hybrid model. By incorporating machine learning algorithms, the model could potentially improve its accuracy in selecting the optimal stock portfolio. This could involve training the model on historical stock data to learn patterns and make more informed portfolio decisions.

2. Risk Management Strategies: The focus could be shifted towards incorporating advanced risk management strategies into the hybrid model. This could involve developing type 2 fuzzy logic-based rules and algorithms to dynamically adjust the portfolio composition based on the changing market conditions and risk factors. The researchers could explore methods to handle various types of risks, such as market risk, credit risk, or liquidity risk, within the framework of the hybrid model.

These suggestions aim to enhance the existing hybrid model by incorporating additional techniques or addressing specific aspects of portfolio optimization. Further research in these directions could potentially lead to improved decision-making capabilities and more robust stock portfolio selection under uncertainty.

## Acknowledgement

The authors wish to express their appreciation for several excellent suggestions for improvements in this paper made by the referees.

Archive of SID.ir

Archive of SID.ir



## References


[1] G. R. Amin, M. Hajjami, *Improving DEA cross-efficiency optimization in portfolio selection*, Expert Systems with Applications, **168** (2021), 114280. https://doi.org/10.1016/j.eswa.2020.114280

[2] K. Atanassov, *Intuitionistic fuzzy sets*, International Journal of Bioautomation, **20** (2016), 1. http://dx.doi.org/10.1016/S0165-0114(86)80034-3

[3] N. K. Avkiran, A. McCrystal, *Sensitivity analysis of network DEA: NSBM versus NRAM*, Applied Mathematics and Computation, **218**(22) (2012), 11226-11239. https://doi.org/10.1016/j.amc.2012.05.014

[4] R. Bagheri, Z. Borouji, S. B. Razavian, M. M. Keshvari, F. Sharifi, S. Sharifi, *Implementation of MCDM-based integrated approach to identifying the uncertainty factors on the constructional project*, Mathematical Problems in Engineering, **2021** (2021), 1-12. https://doi.org/10.1155/2021/1473917

[5] T. T. Bjerring, O. Ross, A. Weissensteiner, *Feature selection for portfolio optimization*, Annals of Operations Research, **256** (2017), 21-40.https://doi.org/10.1007/s10479-016-2155-y

[6] C. Carlsson, R. Fullér, M. Heikkilä, P. Majlender, *A fuzzy approach to R&D project portfolio selection*, International Journal of Approximate Reasoning, **44**(2) (2007), 93-105. https://doi.org/10.1016/j.ijar.2006.07.003

[7] C. Carlsson, R. Fullér, P. Majlender, *A possibilistic approach to selecting portfolios with highest utility score*, Fuzzy Sets and Systems, **131**(1) (2002), 13-21. https://doi.org/10.1016/S0165-0114(01)00251-2

[8] F. Cesarone, A. Scozzari, F. Tardella, *An optimization-diversification approach to portfolio selection*, Journal of Global Optimization, **76**(2) (2020), 245-265. https://doi.org/10.1007/s10898-019-00809-7

[9] T. J. Chang, N. Meade, J. E. Beasley, Y. M. Sharaiha, *Heuristics for cardinality constrained portfolio optimisation*, Computers and Operations Research, **27**(13) (2000), 1271-1302. https://doi.org/10.1016/S0305-0548(99)00074-X

[10] T. S. Chang, K. Tone, C. H. Wu, *Nested dynamic network data envelopment analysis models with infinitely many decision making units for portfolio evaluation*, European Journal of Operational Research, **291**(2) (2021), 766-781. https://doi.org/10.1016/j.ejor.2020.09.044

[11] T. J. Chang, S. C. Yang, K. J. Chang, *Portfolio optimization problems in different risk measures using genetic algorithm*, Expert Systems with Applications, **36**(7) (2009), 10529-10537. https://doi.org/10.1016/j.eswa.2009.02.062

[12] A. Charnes, W. W. Cooper, E. Rhodes, *Measuring the efficiency of decision making units*, European Journal of Operational Research, **2**(6) (1978), 429-444. https://doi.org/10.1016/0377-2217(78)90138-8

[13] W. Chen, S. S. Li, J. Zhang, M. K. Mehlawat, *A comprehensive model for fuzzy multi-objective portfolio selection based on DEA cross-efficiency model*, Soft Computing, **24** (2020), 2515-2526. https://doi.org/10.1007/s00500-018-3595-x

[14] B. Chen, J. D. Zhong, Y. Y. Chen, *A hybrid approach for portfolio selection with higher-order moments: Empirical evidence from Shanghai stock exchange*, Expert Systems with Applications, **145** (2020), 113104. https://doi.org/10.1016/j.eswa.2019.113104

[15] W. W. Cooper, K. S. Park, J. T. Pastor, *RAM: A range adjusted measure of inefficiency for use with additive models, and relations to other models and measures in DEA*, Journal of Productivity Analysis, **11** (1999), 5-42. https://doi.org/10.1023/A:1007701304281

[16] O. V. De la Torre-Torres, E. Galeana-Figueroa, M. Cruz Del Río-Rama, J. Álvarez-García, *Using Markov-switching models in US stocks optimal portfolio selection in a Black-Litterman context (part 1)*, Mathematics, **10**(8) (2022), 1296. https://doi.org/10.3390/math10081296

[17] S. A. Edalatpanah, *A data envelopment analysis model with triangular intuitionistic fuzzy numbers*, International Journal of Data Envelopment Analysis, **7**(4) (2019). https://doi.org/10.1049/trit.2020.0016


Archive of SID.ir

Archive of SID.ir



[18] N. Edirisinghe, P. Chanaka, X. Zhang, *Portfolio selection under DEA-based relative financial strength indicators: Case of US industries*, Journal of the Operational Research Society, **59** (2008), 842-856. https://doi.org/10.1057/palgrave.jors.2602442

[19] R. Färe, S. Grosskopf, *Modeling undesirable factors in efficiency evaluation: Comment*, European Journal of Operational Research, **157**(1) (2004), 242-245. https://doi.org/10.1016/S0377-2217(03)00191-7

[20] M. García-Melón, R. Poveda-Bautista, *Using the strategic relative alignment index for the selection of portfolio projects application to a public Venezuelan power corporation*, International Journal of Production Economics, **170** (2015), 54-66. https://doi.org/10.1016/j.ijpe.2015.08.023

[21] S. Giove, S. Funari, C. Nardelli, *An interval portfolio selection problem based on regret function*, European Journal of Operational Research, **170**(1) (2006), 253-264. https://doi.org/10.1016/j.ejor.2004.05.030

[22] J. P. Gómez, T. Sharma, *Portfolio delegation under short-selling constraints*, Economic Theory, **28** (2006), 173-196. https://doi.org/10.1007/s00199-004-0615-0

[23] P. Grzegorzewski, *Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric*, Fuzzy Sets and Systems, **148**(2) (2024), 319-328. https://doi.org/10.1007/s00199-004-0615-0

[24] H. Guo, B. Sun, H. R. Karimi, Y. Ge, W. Jin, *Fuzzy investment portfolio selection models based on interval analysis approach*, Mathematical Problems in Engineering, **2012** (2012). https://doi.org/10.1155/2012/628295

[25] X. Huang, H. Di, *Uncertain portfolio selection with background risk*, Applied Mathematics and Computation, **276** (2016), 284-296. https://doi.org/10.1016/j.amc.2015.12.018

[26] M. Khedmati, P. Azin, *An online portfolio selection algorithm using clustering approaches and considering transaction costs*, Expert Systems with Applications, **159** (2020), 113546. https://doi.org/10.1016/j.eswa.2020.113546

[27] J. D. Lamb, K. H. Tee, *Resampling DEA estimates of investment fund performance*, European Journal of Operational Research, **223**(3) (2012), 834-841. https://doi.org/10.1016/j.ejor.2012.07.015

[28] T. León, V. Liern, E. Vercher, *Viability of infeasible portfolio selection problems: A fuzzy approach*, European Journal of Operational Research, **139**(1) (2002), 178-189. https://doi.org/10.1016/S0377-2217(01)00175-8

[29] Y. Li, Q. Cui, *Carbon neutral growth from 2020 strategy and airline environmental inefficiency: A network range adjusted environmental data envelopment analysis*, Applied Energy, **199** (2017), 13-24. https://doi.org/10.1016/j.apenergy.2017.04.072

[30] Z. Mashayekhi, H. Omrani, *An integrated multi-objective Markowitz–DEA cross-efficiency model with fuzzy returns for portfolio selection problem*, Applied Soft Computing, **38** (2016), 1-9. https://doi.org/10.1142/S0219622014500059

[31] M. K. Mehlawat, P. Gupta, *Credibility-based fuzzy mathematical programming model for portfolio selection under uncertainty*, International Journal of Information Technology and Decision Making, **13**(01) (2014), 101-135. https://doi.org/10.1142/S0219622014500059

[32] M. K. Mehlawat, A. Kumar, S. Yadav, W. Chen, *Data envelopment analysis based fuzzy multi-objective portfolio selection model involving higher moments*, Information Sciences, **460** (2018), 128-150. https://doi.org/10.1016/j.ins.2018.05.043

[33] S. Moslemi, A. Mirzazadeh, *Performance evaluation of four-stage blood supply chain with feedback variables using NDEA cross-efficiency and entropy measures under IER uncertainty*, Numerical Algebra, Control and Optimization, **7**(4) (2017), 379-401. https://doi.org/10.3934/naco.2017024

[34] M. Nourahmadi, H. Sadeqi, *A machine learning-based hierarchical risk parity approach: A case study of portfolio consisting of stocks of the top 30 companies on the Tehran stock exchange*, Financial Research Journal, **24**(2) (2022), 236-256. https://doi.org/10.22059/frj.2021.319092.1007146

[35] H. Omrani, Z. Mashayekhi, *Multi objective portfolio selection by combining markowitz and dea cross efficiency models*, Sharif Journal of Industrial Engineering and Management, **33**(1.1) (2017), 87-94. https://doi.org/10.24200/j65.2017.5578

Archive of SID.ir

Archive of SID.ir



[36] R. Pal, T. D. Chaudhuri, S. Mukhopadhyay, *Portfolio formation and optimization with continuous realignment: A suggested method for choosing the best portfolio of stocks using variable length NSGA-II*, Expert Systems with Applications, **186** (2021), 115732. https://doi.org/10.1016/j.eswa.2021.115732

[37] J. L. Pedersen, G. Peskir, *Optimal mean-variance portfolio selection*, Mathematics and Financial Economics, **11** (2017), 137-160. https://doi.org/10.1007/s11579-016-0174-8

[38] G. Peralta, A. Zareei, *A network approach to portfolio selection*, Journal of Empirical Finance, **38** (2016), 157-180. https://doi.org/10.1016/j.jempfin.2016.06.003

[39] A. R. Pouya, M. Solimanpur, M. Jahangoshai Rezaee, *Solving multi-objective portfolio optimization problem using invasive weed optimization*, Swarm and Evolutionary Computation, **28** (2016), 42-57. https://doi.org/10.1016/j.swevo.2016.01.001

[40] J. Puri, S. Prasad Yadav, *Intuitionistic fuzzy data envelopment analysis: An application to the banking sector in India*, Expert Systems with Applications, **42**(11) (2015), 4982-4998. https://doi.org/10.1016/j.eswa.2015.02.014

[41] M. Rasoulzadeh, S. A. Edalatpanah, M. Fallah, S. E. Najafi, *A multi-objective approach based on Markowitz and DEA cross-efficiency models for the intuitionistic fuzzy portfolio selection problem*, Decision Making: Applications in Management and Engineering, **5**(2) (2022), 241-259. https://doi.org/10.31181/dmame0324062022e

[42] A. M. Rather, V. N. Sastry, A. Agarwal, *Stock market prediction and Portfolio selection models: A survey*, Opsearch, **54** (2017), 558-579. https://doi.org/10.1007/s12597-016-0289-y

[43] I. B. Salehpoor, S. Molla-Alizadeh-Zavardehi, *A constrained portfolio selection model at considering risk-adjusted measure by using hybrid meta-heuristic algorithms*, Applied Soft Computing, **75** (2019), 233-253. https://doi.org/10.1016/j.asoc.2018.11.011

[44] M. Shadabfar, L. Cheng, *Probabilistic approach for optimal portfolio selection using a hybrid Monte Carlo simulation and Markowitz model*, Alexandria Engineering Journal, **59**(5) (2020), 3381-3393. https://doi.org/10.1016/j.aej.2020.05.006

[45] A. H. Tajik Yabr, S. E. Najafi, Z. Moghaddas, P. Shahnazari Shahrezaei, *Interval cross efficiency measurement for general two-stage systems*, Mathematical Problems in Engineering, **2022** (2022). https://doi.org/10.1155/2022/5431358

[46] B. Wang, Y. Li, S. Wang, J. Watada, *A multi-objective portfolio selection model with fuzzy value-at-risk ratio*, IEEE Transactions on Fuzzy Systems, **26**(6) (2018), 3673-3687. https://doi.org/10.1109/TFUZZ.2018.2842752

[47] S. Wang, S. Zhu, *On fuzzy portfolio selection problems*, Fuzzy Optimization and Decision Making, **1** (2002), 361-377. https://doi.org/10.1023/A:1020907229361

[48] J. Watada, *Fuzzy portfolio selection and its applications to decision making*, Tatra Mountains Mathematical Publication, **13** (1997), 219-248. http://dx.doi.org/10.1007/978-3-7908-1817-8_7

[49] A. Yoshimoto, *The mean-variance approach to portfolio optimization subject to transaction costs*, Journal of the Operations Research Society of Japan, **39**(1) (1996), 99-117. https://doi.org/10.15807/jorsj.39.99

[50] X. Zhou, J. Wang, X. Yang, B. Lev, Y. Tu, S. Wang, *Portfolio selection under different attitudes in fuzzy environment*, Information Sciences, **462** (2018), 278-289. https://doi.org/10.1016/j.ins.2018.06.013

[51] W. Zhou, Z. Xu, *Portfolio selection and risk investment under the hesitant fuzzy environment*, Knowledge-Based Systems, **144** (2018), 21-31. https://doi.org/10.1016/j.knosys.2017.12.020

Archive of SID.ir